PENN WHARTON
UNIVERSITY of PENNSYLVANIA

Budget Model

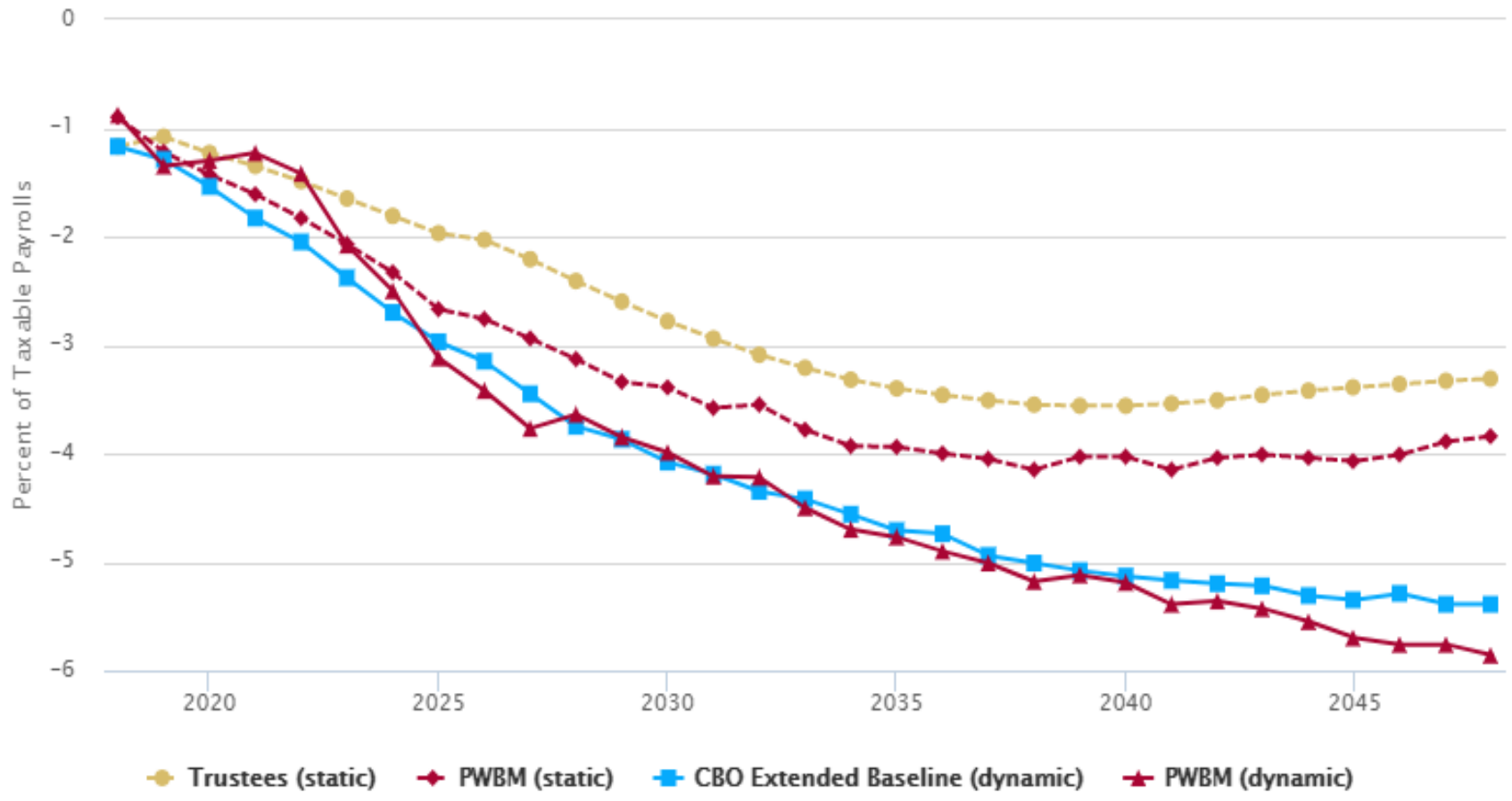
Introduction to PWBM Microsimulation with Application to Social Security

Presentation to Social Security Technical Panel, by Jagadeesh Gokhale

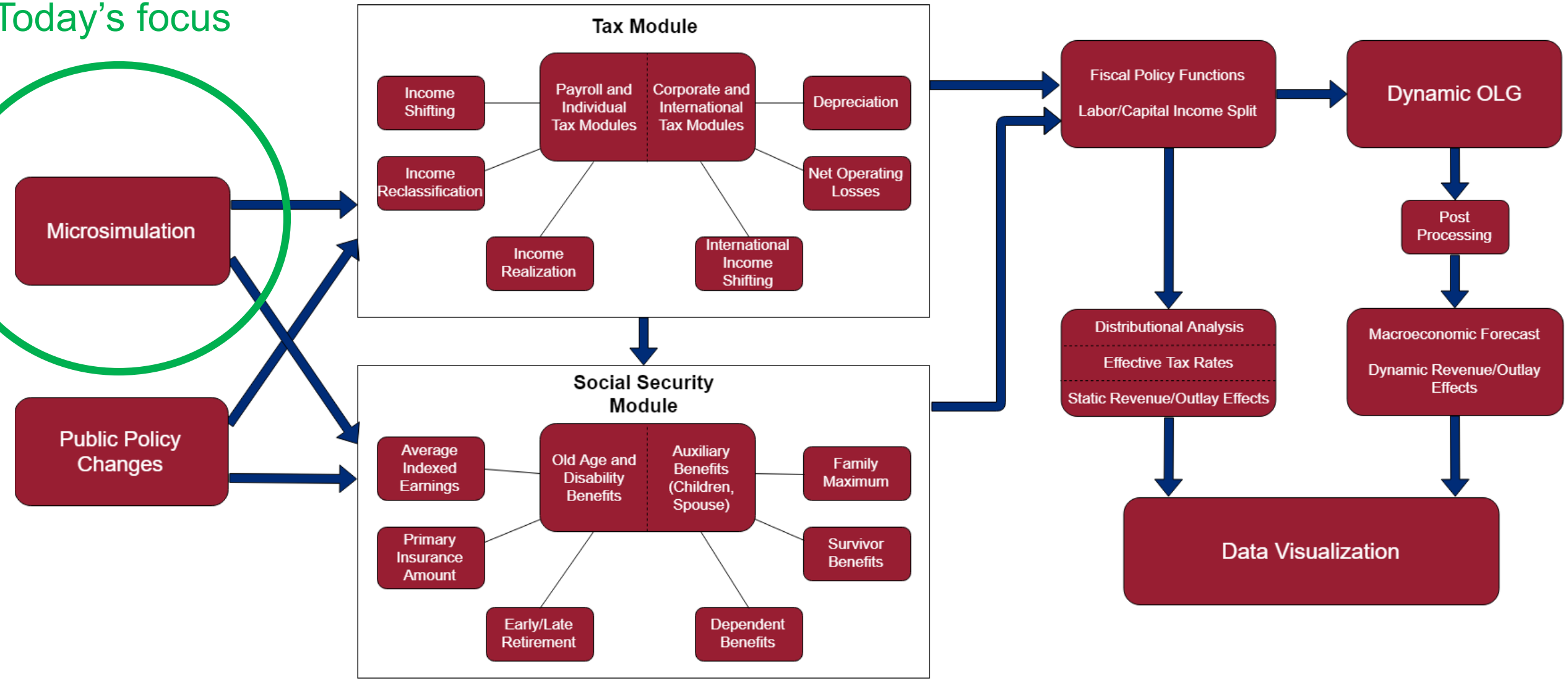
February, 2019

Summary

Figure 4: Social Security (OASDI) Projected Annual non-Interest Balance Ratios (percent)



PWBM Social Security Module



Policies

OASDI Tax Rate

12.4% Progressive Increase 13.6% 14.9%

Taxable Maximum Policy

$128,700 $150,000

Payroll Taxes on Wage Earnings Above $250,000

None 1/2 OASDI Rate Full OASDI Rate

Cost of Living Adjustment (COLA) Index

CPI Elderly CPI Chained CPI

Primary Insurance Amount (PIA)

90/32/15 90/25/8 80/22/5

Full-Benefit Retirement Age (FRA)

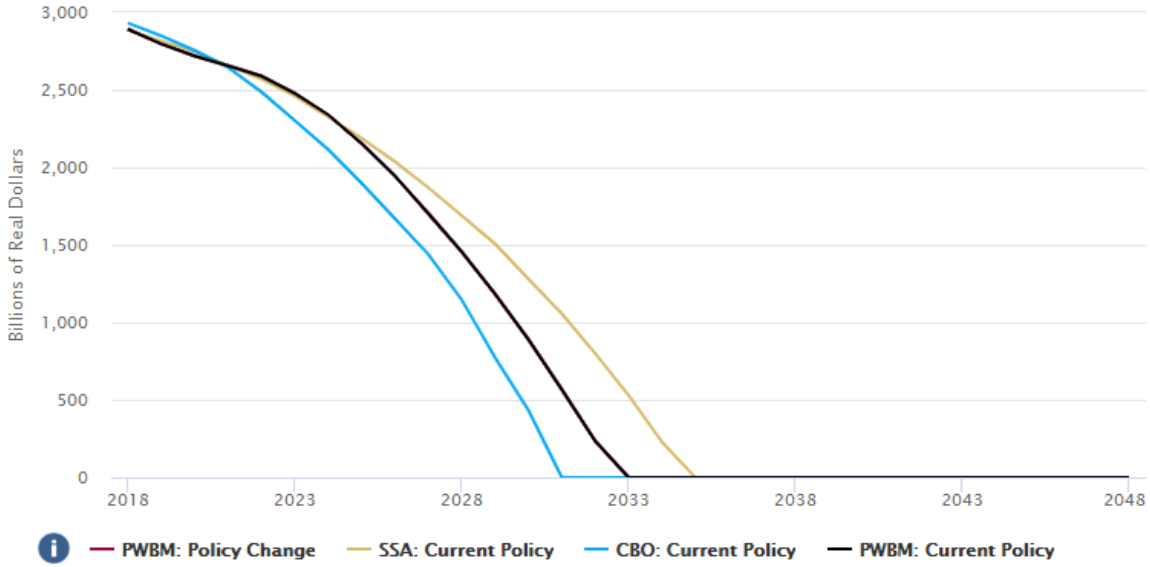
67 68 70

Key Assumptions

SOCIAL SECURITY

Trust Fund Reserves

Download



The Social Security Trust Fund consists of excess revenues from Social Security taxes, which are invested in non-marketable Treasury securities. Interest income from these securities that is not used to pay benefits is also deposited in the Trust Fund.

The real (inflation-adjusted) value of the Trust Fund is expected to decline over time. Even during the next few years, when the difference between Social Security's interest income and its non-interest revenue shortfall will be positive, deposits into the Trust Fund will be insufficient to offset losses in its real value because of inflation. As the difference between program costs and income from Social Security taxes grows larger, interest income will eventually be insufficient to cover the non-interest revenue shortfall. Trust Fund securities would then have to be redeemed to pay lawful benefits, accelerating the decline in the Trust Fund's real value.

More information about the Penn Wharton Budget Model's Social Security simulator.

Policy Brief summarizing findings about Social Security's Financial Condition.

More information about the ranges and default settings for the policy simulator dial controls.

NOTE: Constant dollars are for 2018 base year.

Motivating Microsimulation

Must be explicit:

- … of assumptions (exogenous), equations (endogenous) and relationships.
- … your view of the economy's production side that generates the wage base.
- Can easily identify additional room for improvement of the model.

Therefore:

- Can back-test model on historical data to see how well functional relationships worked historically before projecting forward. We validate against 50+ parametric and non-parametric validations.
- Can easily generate uncertainty (e.g., confidence intervals) within a model consistent way without just letting things run loose or setting everything to "optimistic" or "pessimistic" in a non-model consistent manner (e.g., interest rates inconsistent with assumed growth).

“Future is Different than the Past”

Non-microsim actuarial approach:

- Some actuarial estimates done by age-sex (e.g., labor-force participation, mortality)
- Some estimates apply a growth number to entire labor force (productivity), SR vs. LR.

Example: Productivity

- Without microsim: Replacing a highly productive person going into retirement with a young person has no impact on tax base.
- With microsim: *Conditioning* on many attributes <age, gender, education, race, …>, productivity grows linear over time. Hence, *unconditional* productivity grows non-linear over time (see below).

Of course, one can add a time-indexed “add-factor” to the non-microsim actuarial model, but one needs to first run the microsim model to figure it out.

Let’s Dig In …

Individuals and Families: General Simulation Sequence

0	SIM Start	Initial population from ASEC with all attributes as of Dec. 31, 1996 aged 0-120
1	Aging	Add 1 year to each person's age: Age 0:119 --> ages 1:120; $A_t = A_{t-1} + 1$
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5	Education	Age 6+ advance education years: $p(\Delta e) = p(e_{t-1}\|age_t, ethn, gender)$
6	Disability	People 0+ transit in-out of work impairment status (not SSDI): $\delta_t = d(\delta_{t-1}\|age, sex, ethn)$
7	LFP and FTE work hours	People 18+ through FTE weeks employed (0-104): $\omega_t = \omega(\omega_{t-1}\|age_t, sex, ethn, \ldots many)$
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12	Calculators	Wages and S. E. income Capital assigner (calibrated to BLS estimates of capital services) Benefits calculators: Social Security (other transfers under development) Tax calculators (Individual Income, Payroll, and Corporate)
13	Dynamic	GE-OLG Model with heterogeneous agents – attributes calibrated from SIM

While education is a good proxy for income, future work will also condition on income and iterate between micro-sim and dynamic Gauss-Seidel style until convergence.

Will discuss these in next slides in more detail as examples.

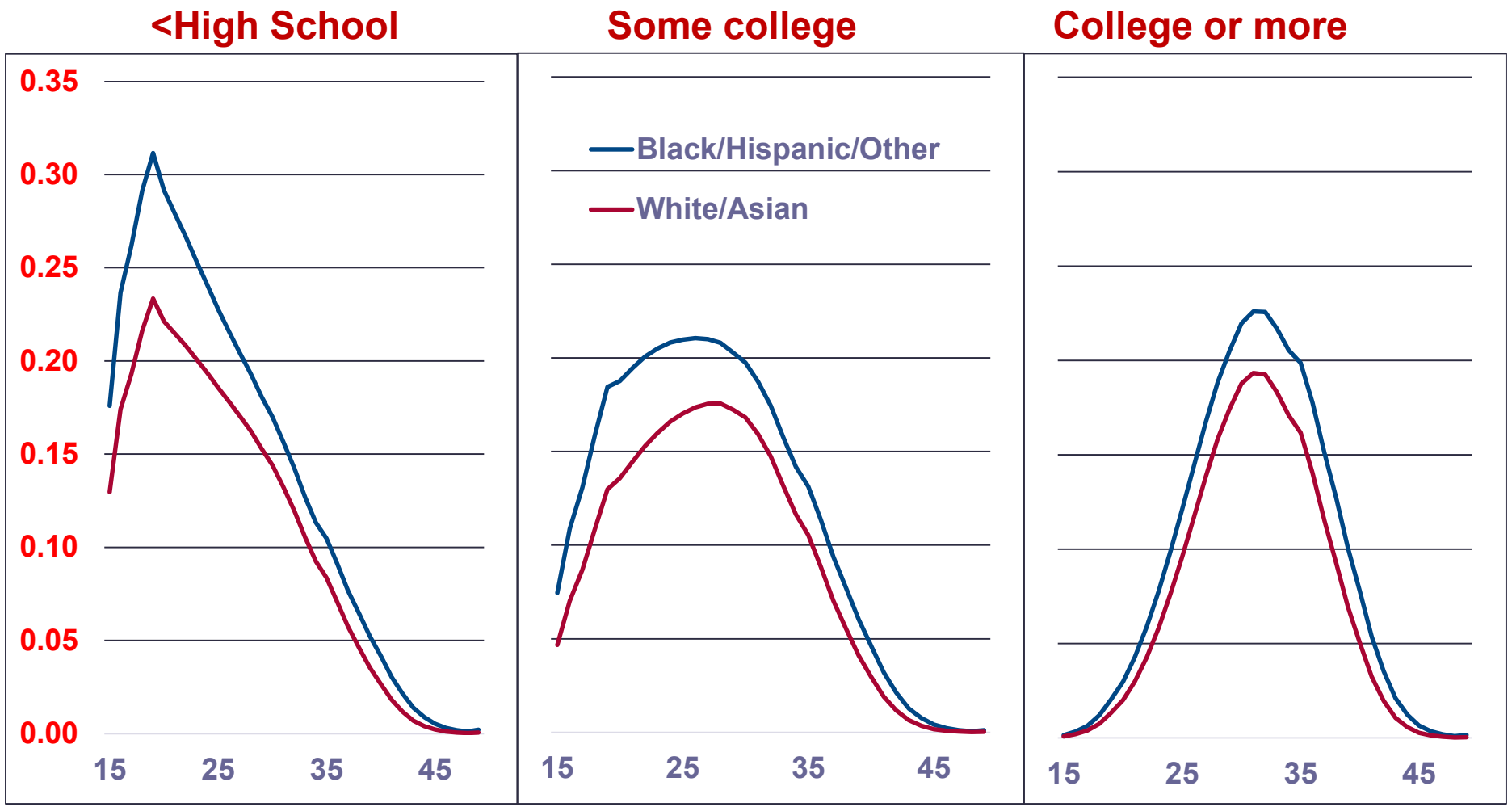
Microsimulation Example: Fertility (Simplified Explanation)

Step 1: Limited dependent regression on historical data (1996-2011) of female attributes per age (race, #children, education, marital status) to create 3x2x3x2=36 ratios of relative differences from mean, per age. Include time dummies intercepts to soak up unexplained variation by year. (Coefficients are not over-fitted, i.e., year independent.)

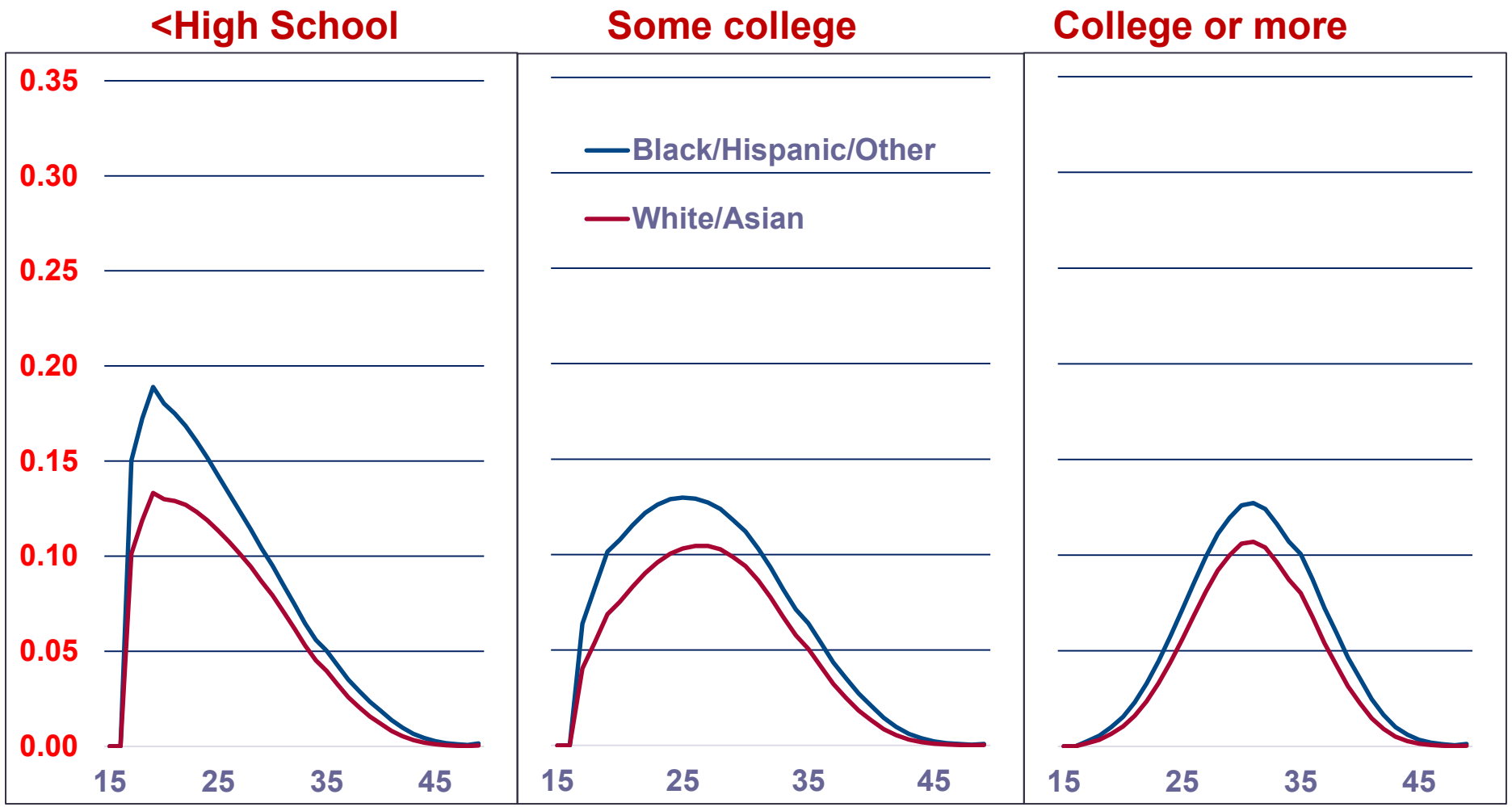
Step 2: Start with 1996 population of females ages 14 – 49, simulate births probabilistically by attribute. Simulated CDF ~ population.

Step 3: Project forward probabilistically with time-varying gradient shift through 2040, using SSA OACT model (that is only conditional on age) or other time gradient source that allows for more conditioning.

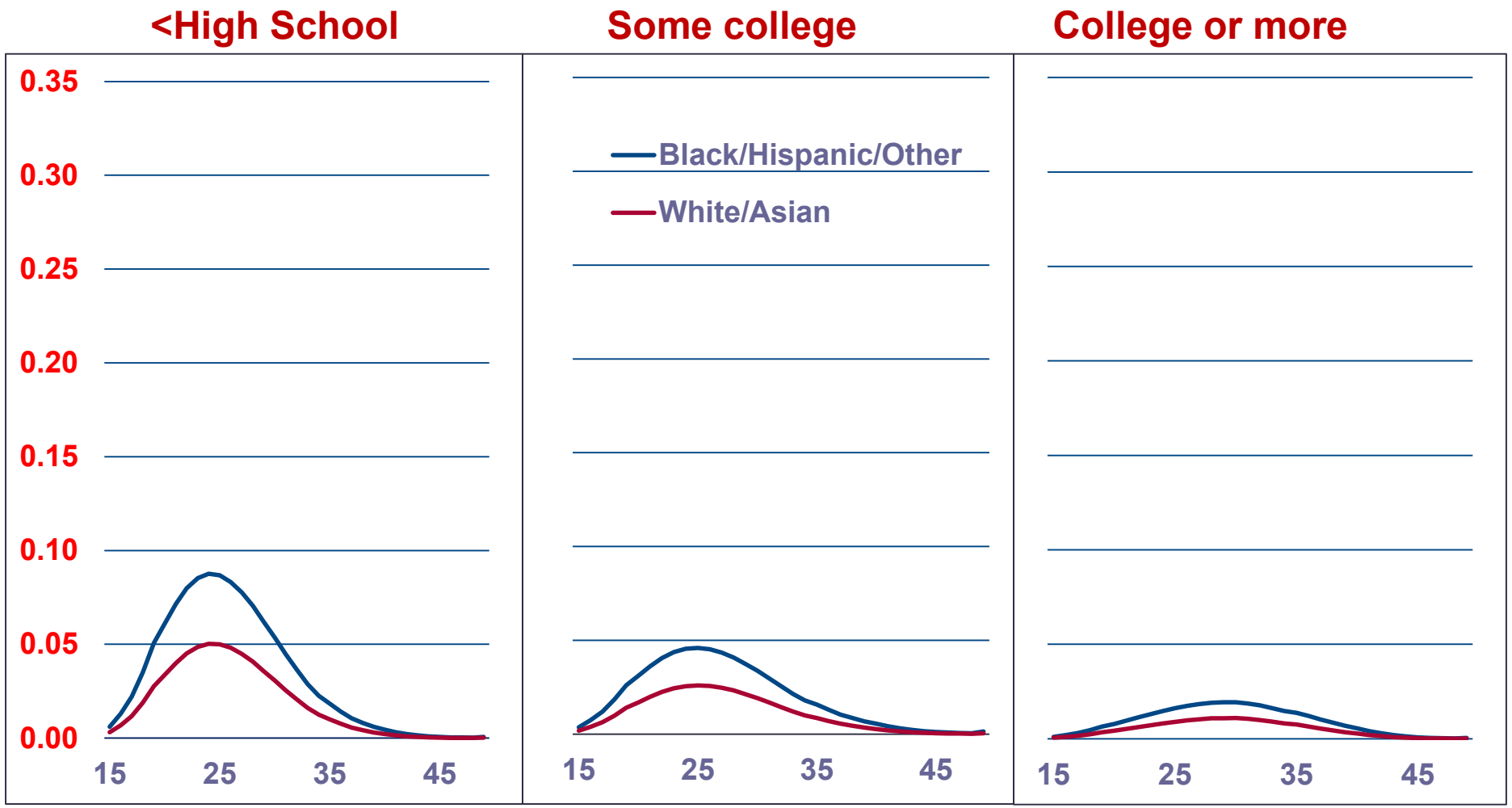
Fitted Fertility Rates by Education: #Kids=0, Married, Year=2015



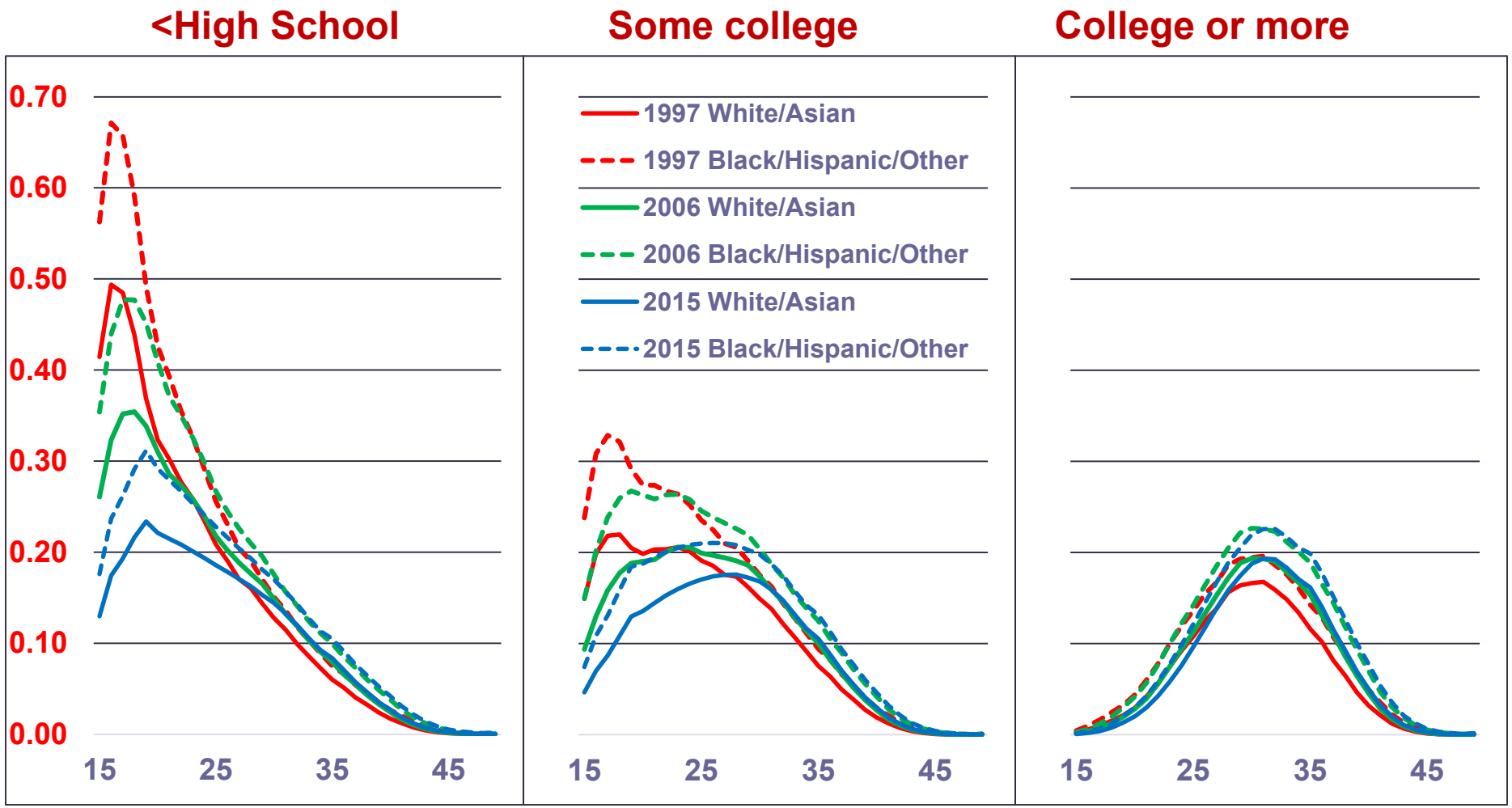
Fitted Fertility Rates by Education for #Kids=2+, Married, Year=2015



Fitted Fertility Rates by Education for #Kids=0, Unmarried, Year=2015



Fitted Fertility Rates by Education for #Kids=0, Married, Years=1997, 2006, 2015



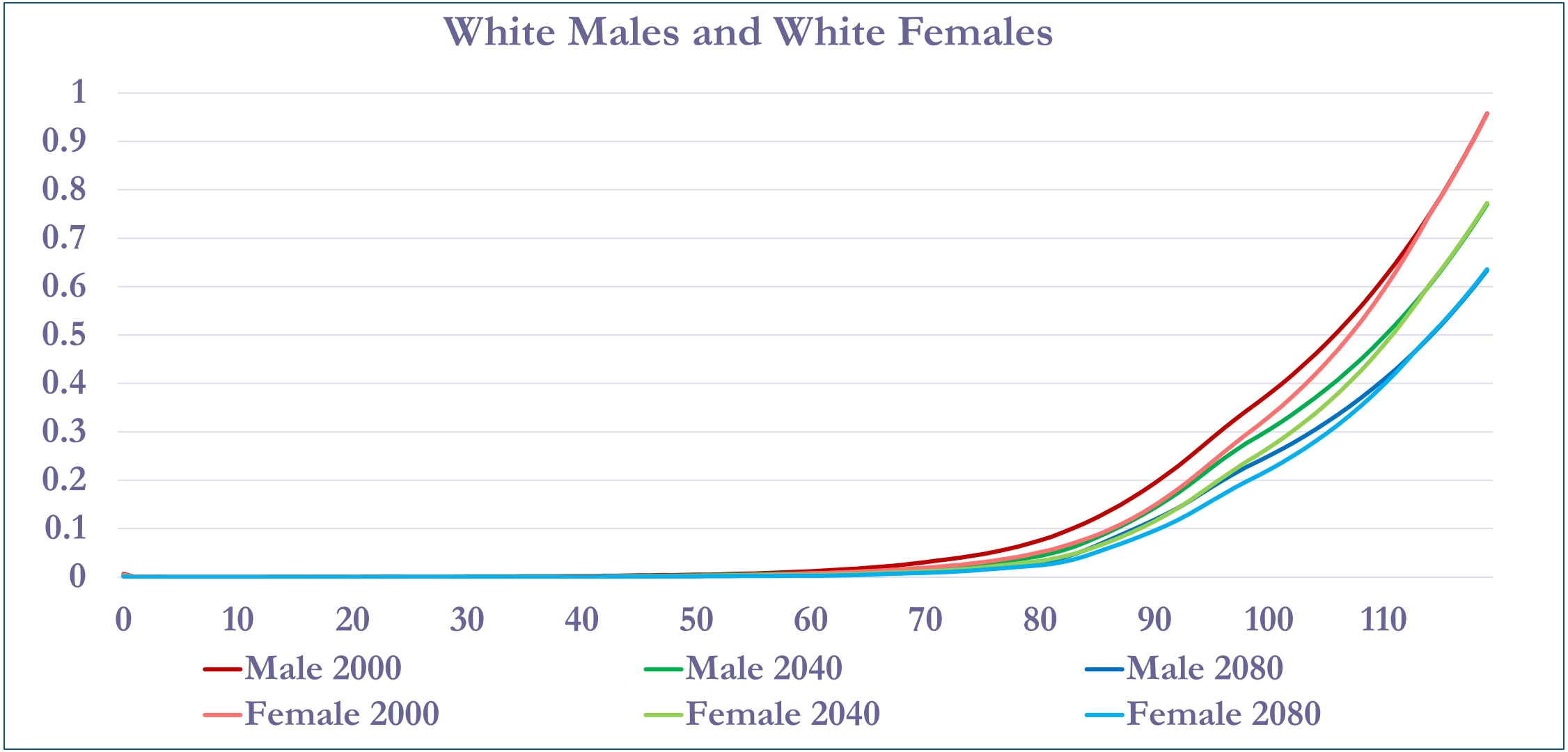
Microsimulation Example: Mortality (Simplified Explanation)

Step 1: Published death rates (NCHS tables,1999-2012) by age, sex, and race are further decomposed by education and marital status using differentials published in the academic literature. This provides 50 ratios of relative differences that are applied to historical average mortality per age and sex.

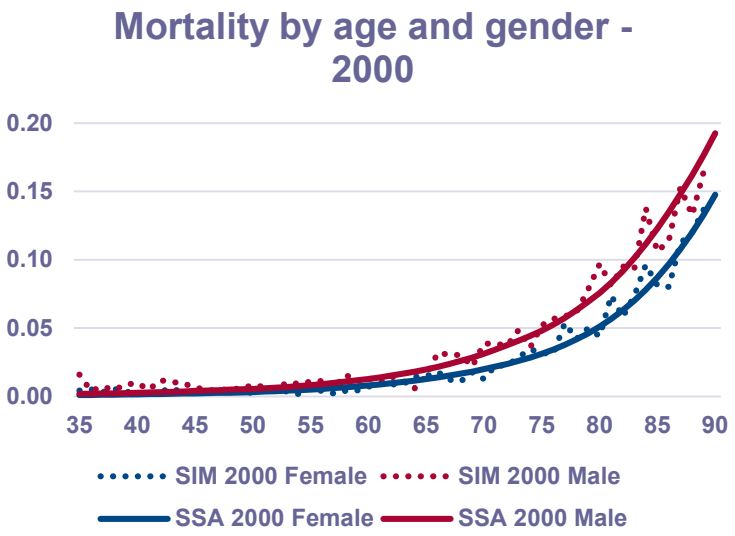
Step 2: Start with 1996 population of individuals aged 0 – 120, simulate deaths probabilistically by attribute. Simulated CDF ~ population.

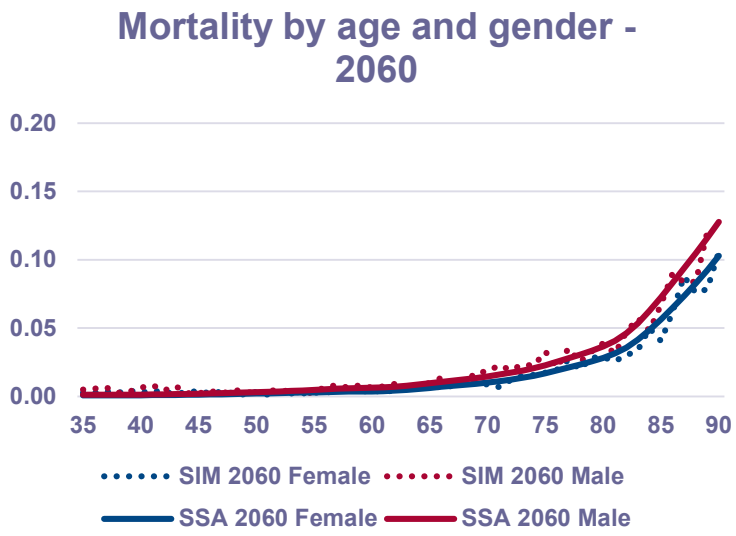
Step 3: Project forward probabilistically with time-varying gradient shift through 75 years, using SSA OACT model (that is only conditional on age and sex) or other gradient source that allows for more conditioning.

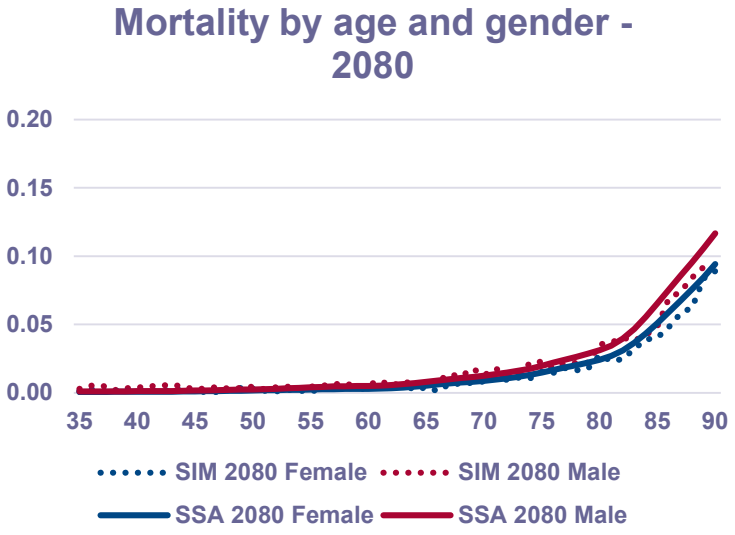
Projected Mortality Target

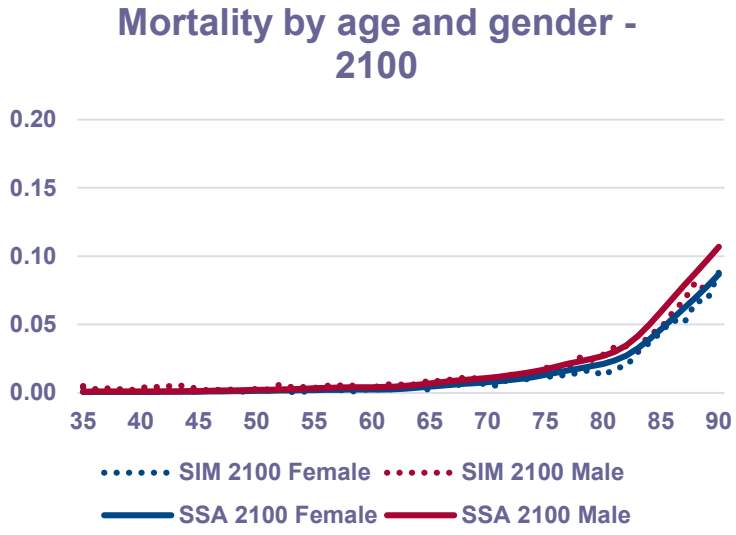


Example: Effect of distinguishing mortality rates by ethnicity



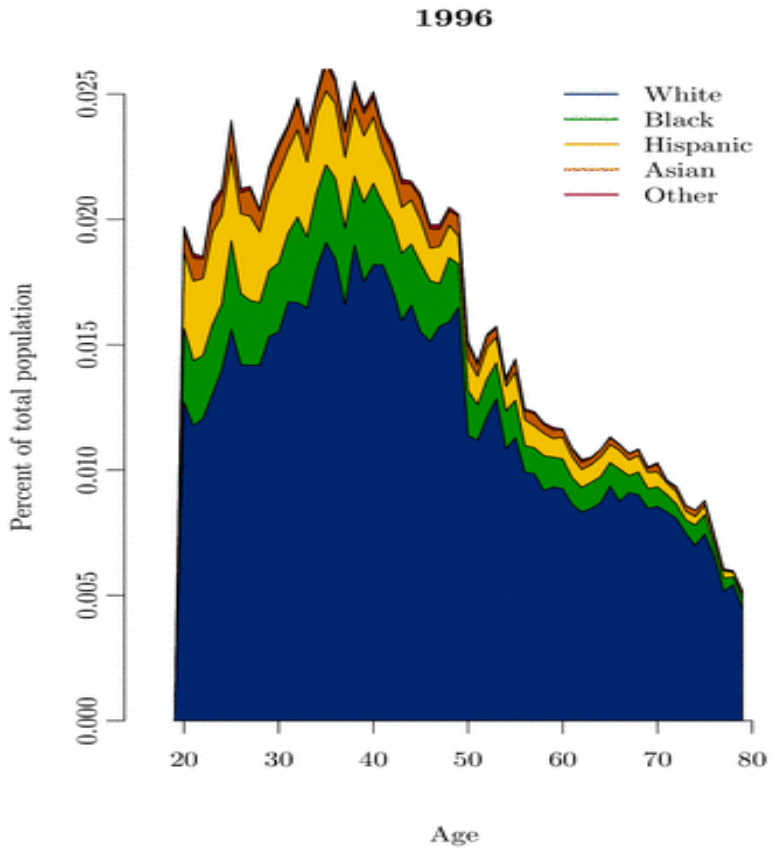




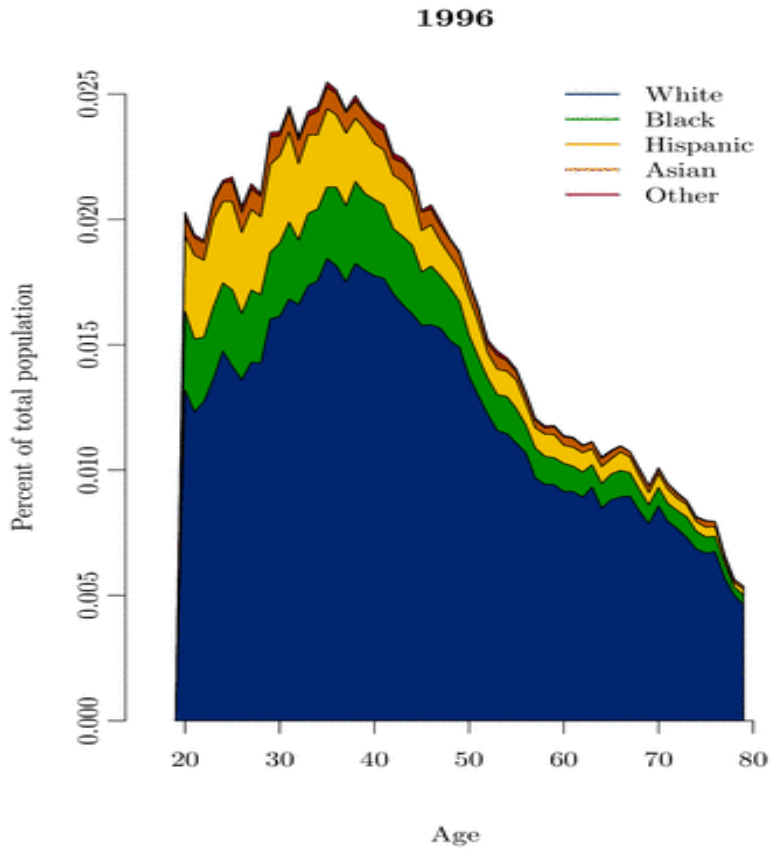


Race / Ethnicity (1996 – 2050)

Census Data



Microsimulation



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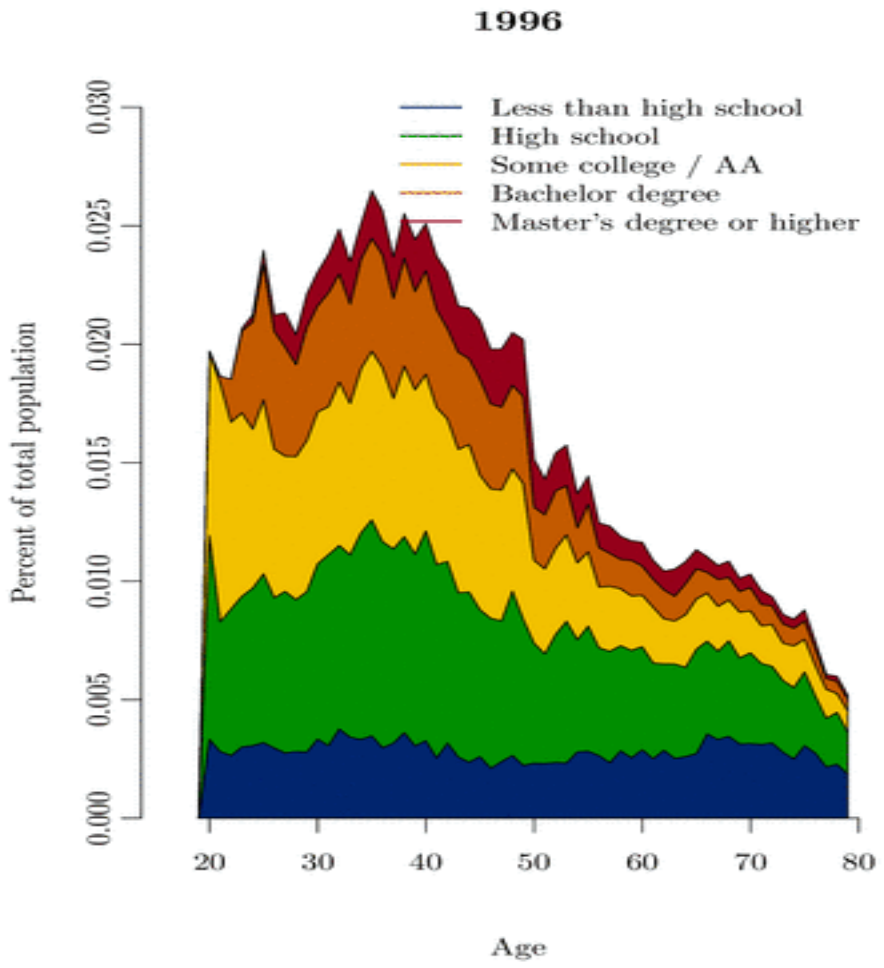
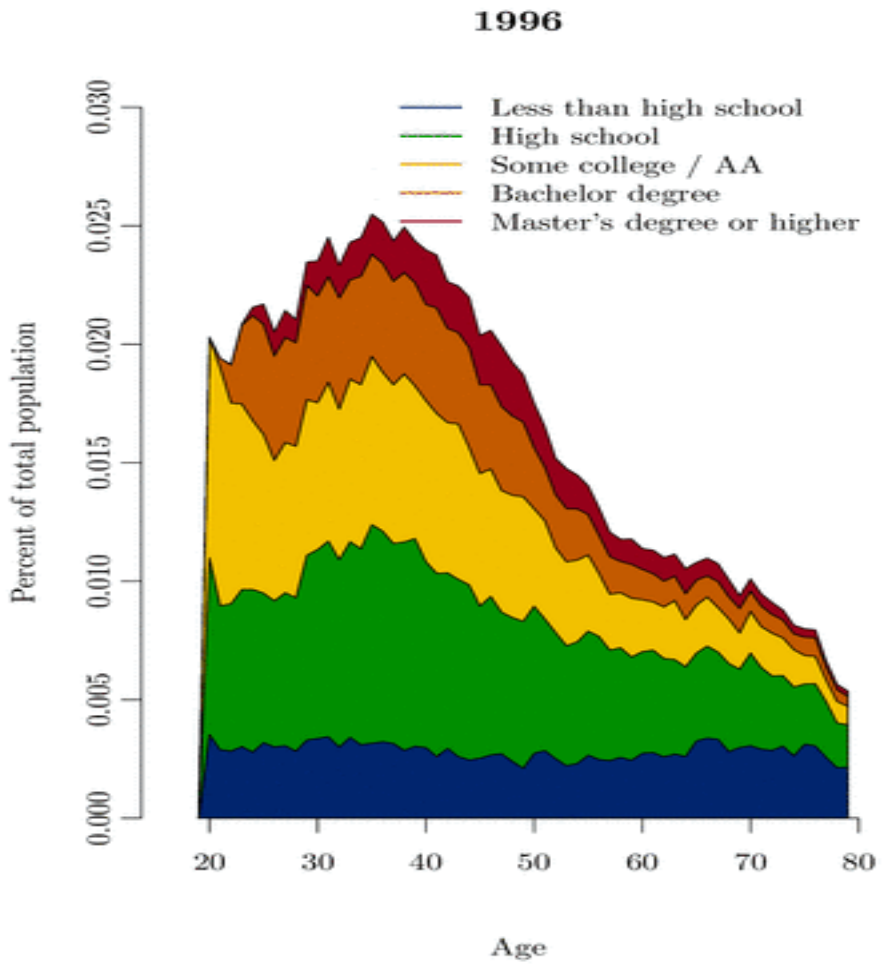
Discussed in more detail on PWBM website. Will just review model outputs for now.

Education (1996 – 2070)

Census Data

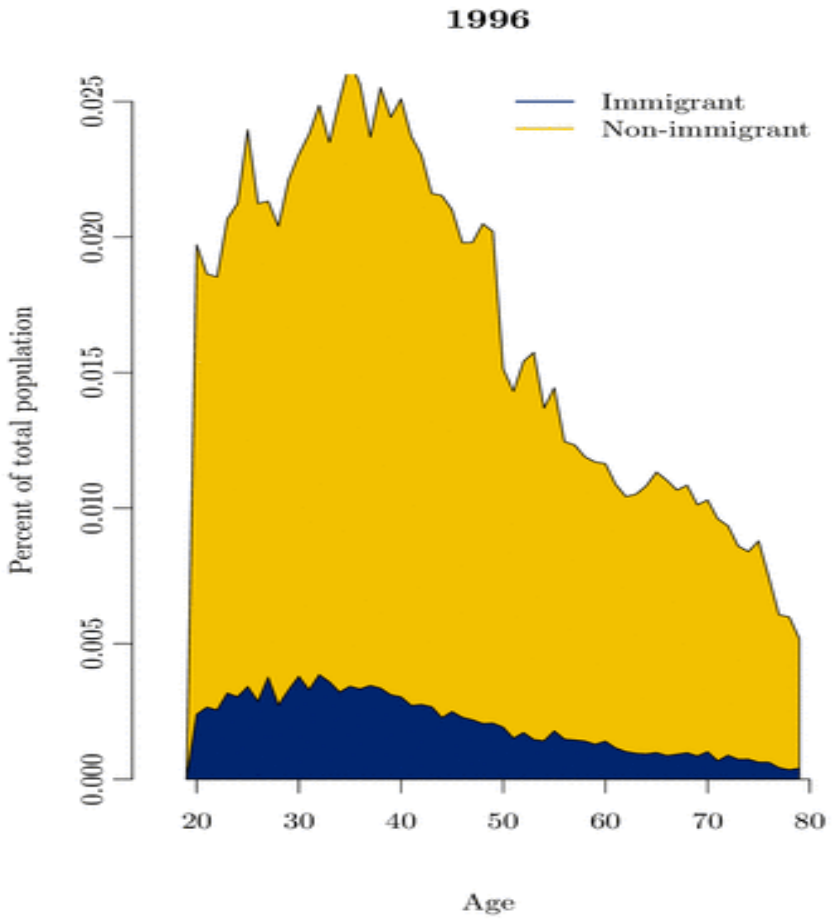


Microsimulation

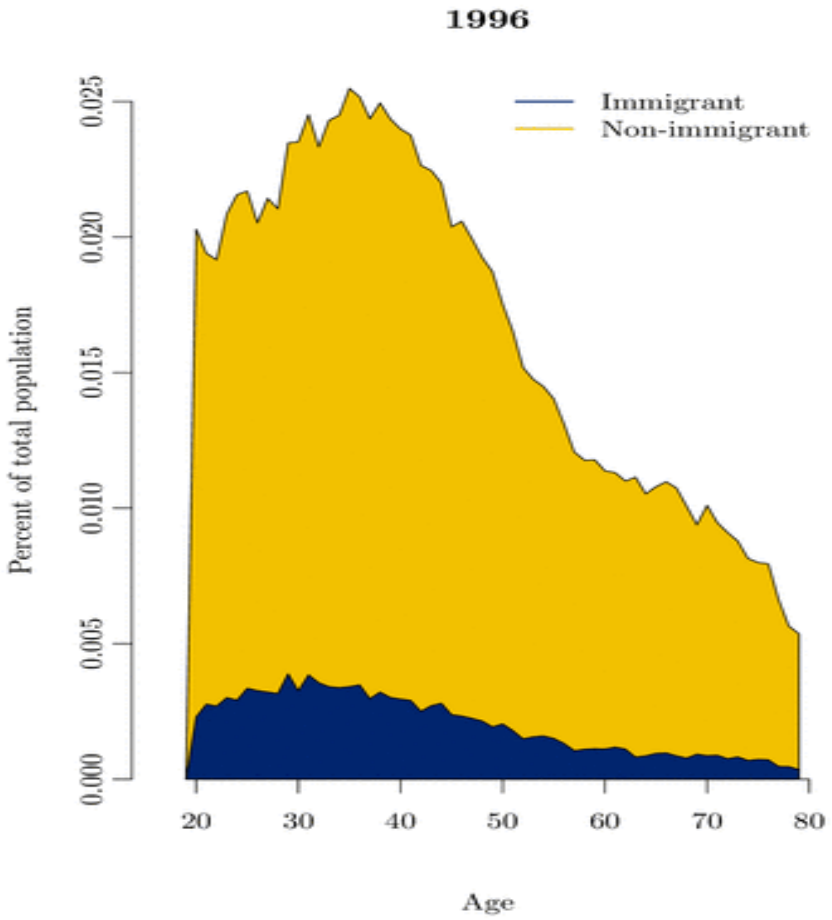


Immigrant Status (1996 – 2050)

Census Data

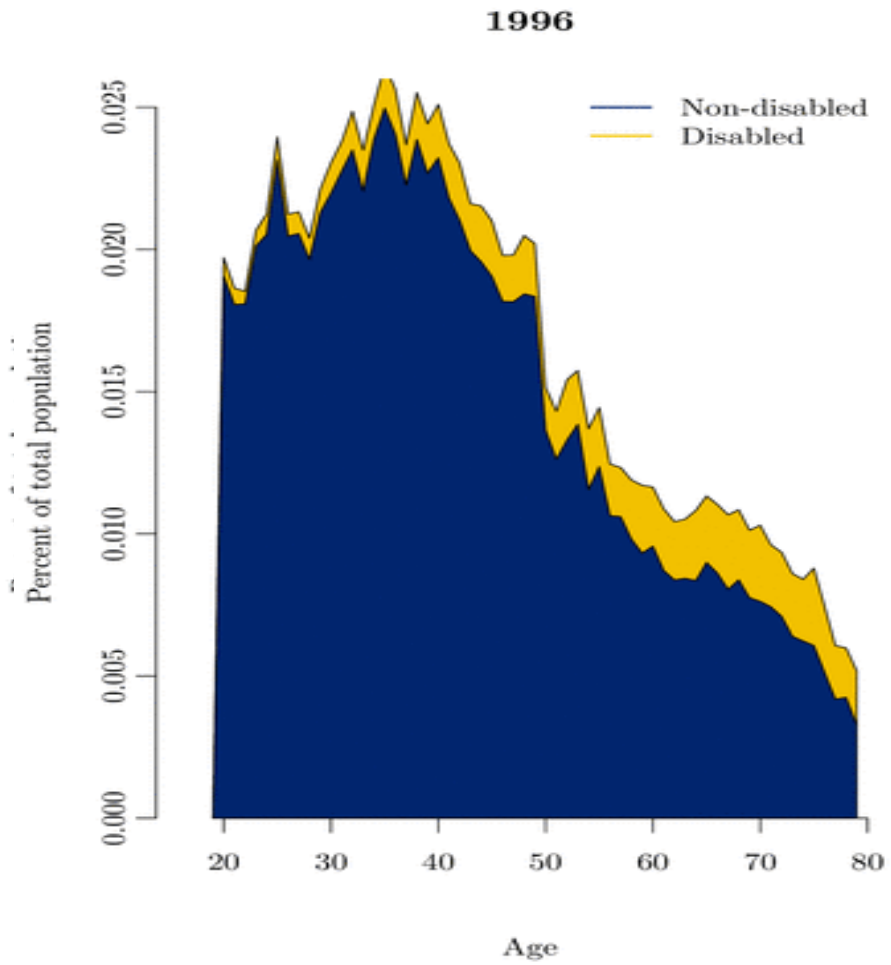


Microsimulation



Disability (1996 – 2070)

Census Data



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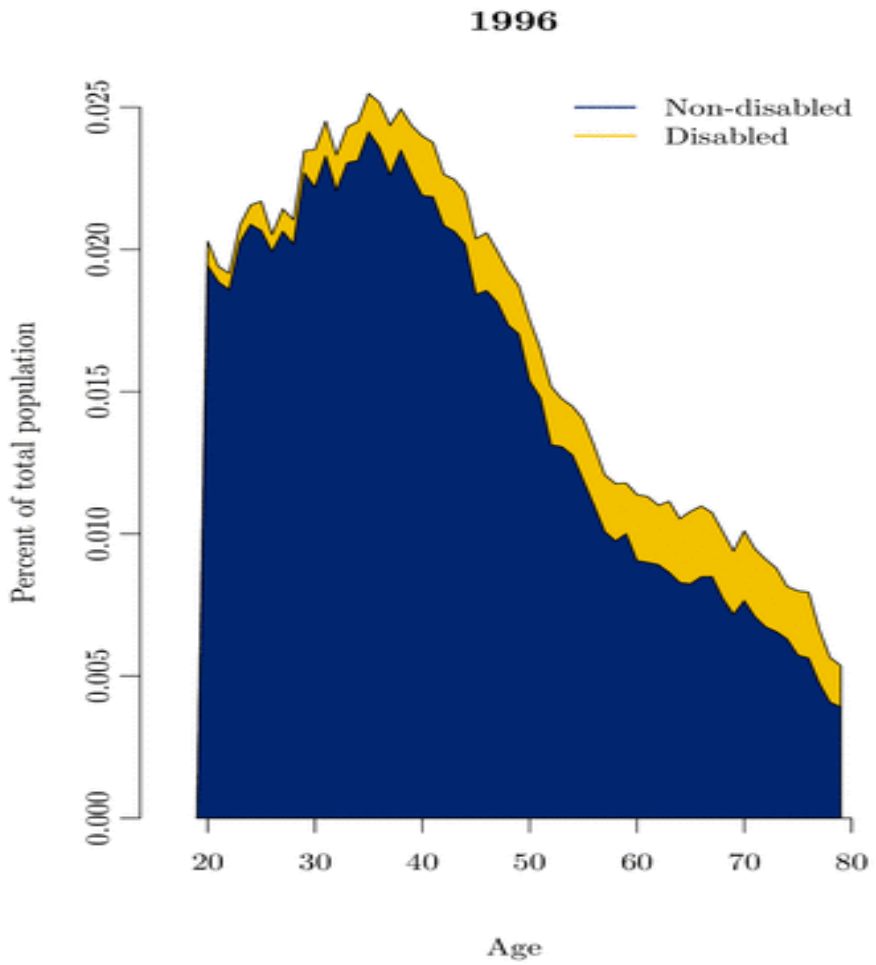
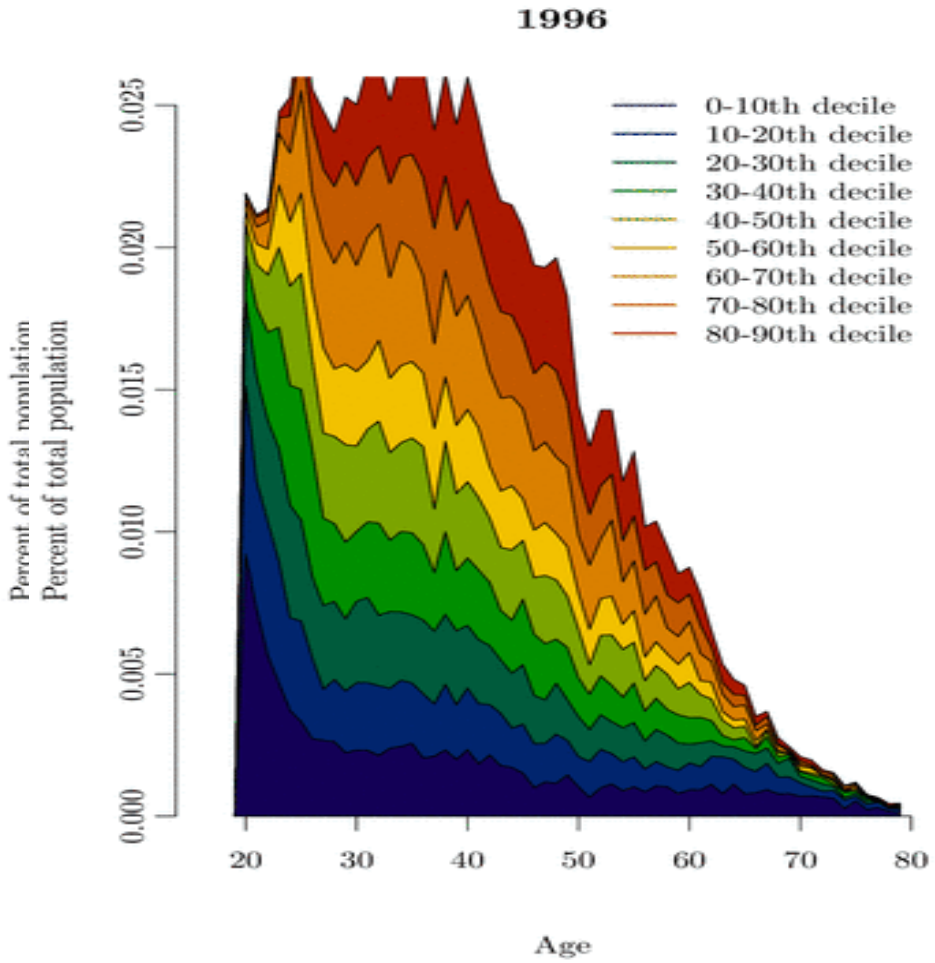
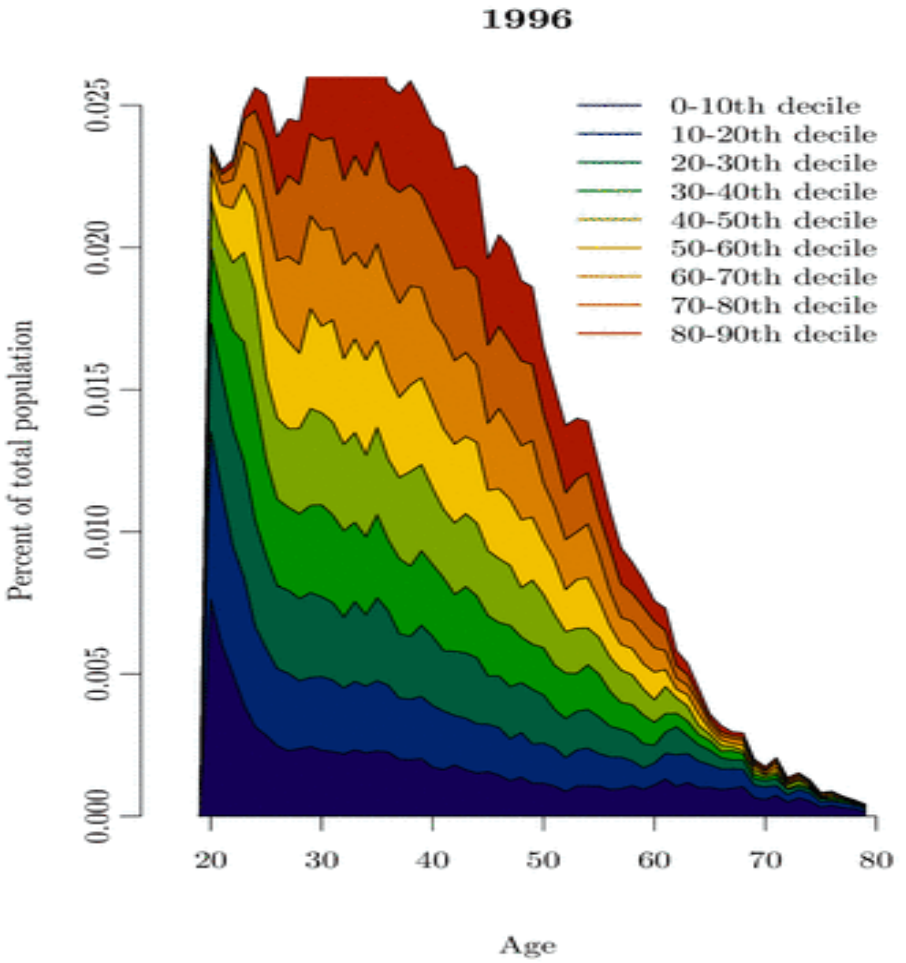


Wage income deciles (1996 – 2070)

Census Data



Microsimulation



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Relationships come from a structural dynamic programming model, given their importance.

Microsimulation: Marriage (Very Brief)

Structural DP model of marriage and divorce (Sophie Shin dissertation)

Existing micro-datasets on *new marriages* are inadequate – use stocks

Person types by initial age and 15 (5 race x 3 education) categories $\{a, r, e\}$

Measure annual change in the stocks of $\{a, r, e\}$ marriages and subtract marriage dissolutions from divorce and death

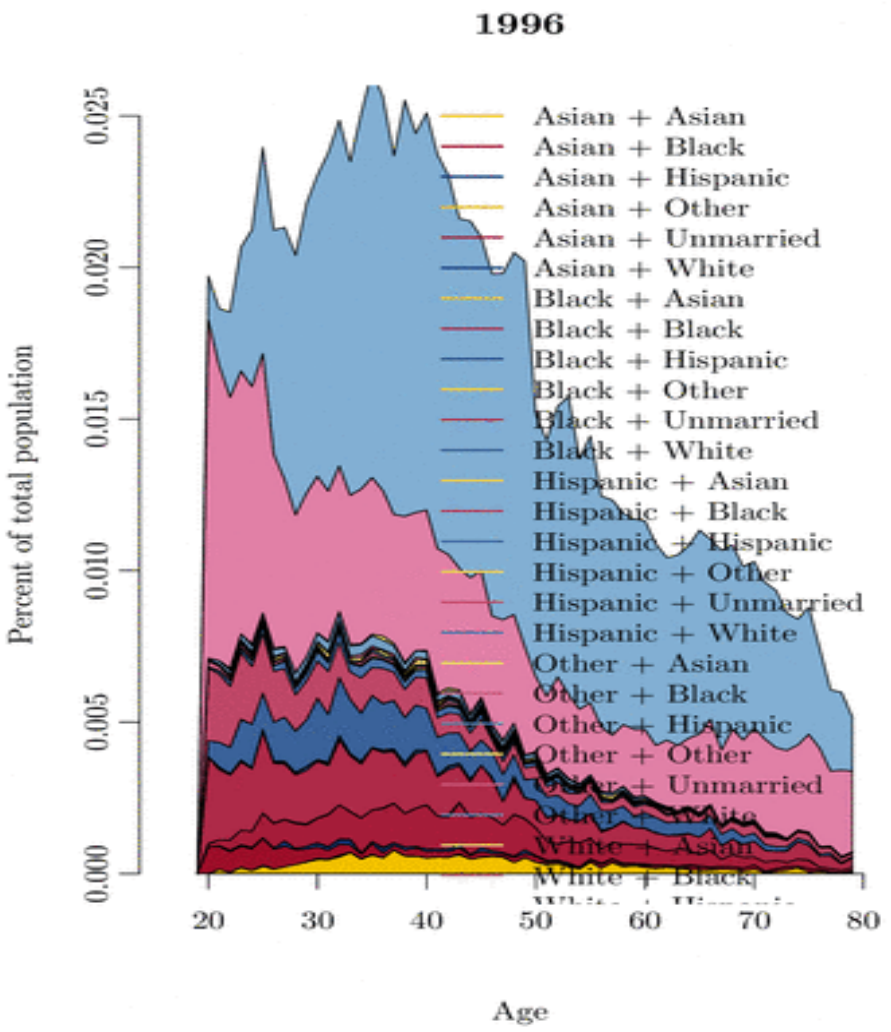
Simulation: Marriages structured in two stages – Meeting and Acceptance

- Meeting rates: x percent within race, (1-x) percent cross-race (including same race)
- Acceptance rates based on match quality – multiplicative weight on comparable age and education levels

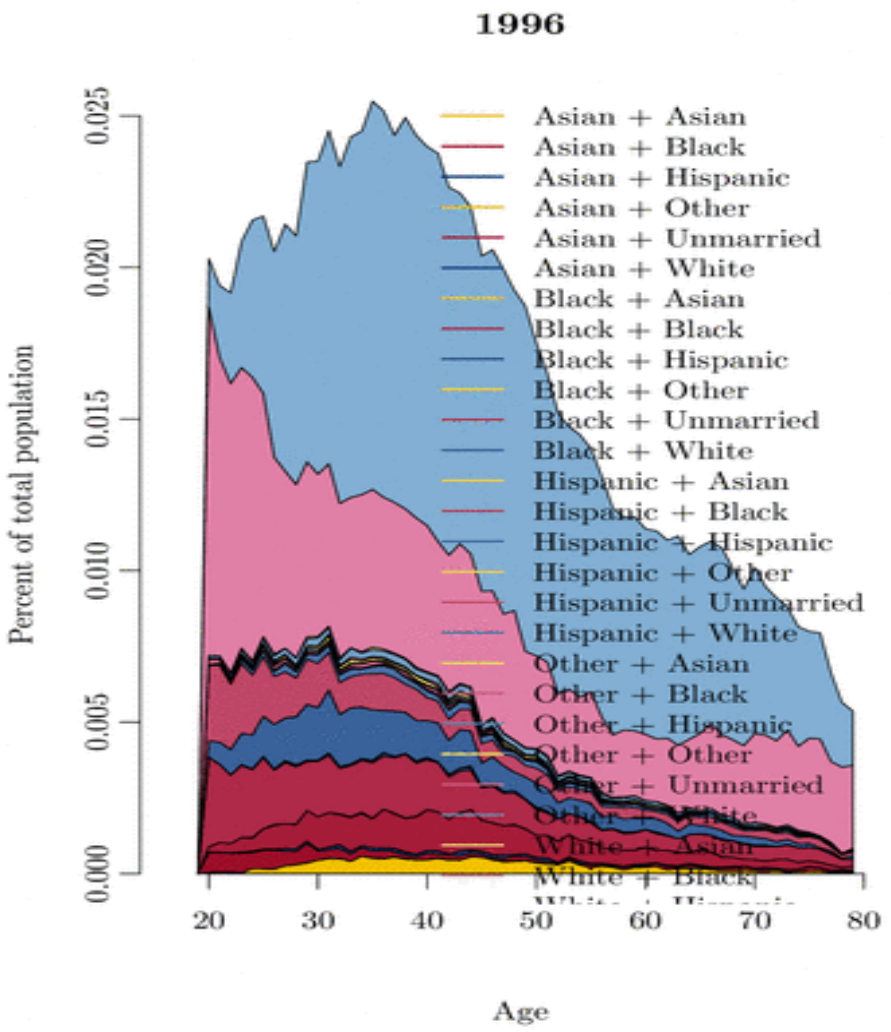
  → assortative pairing by age and education as observed in micro-data

Assortative Mating (1996 – 2050)

Census Data

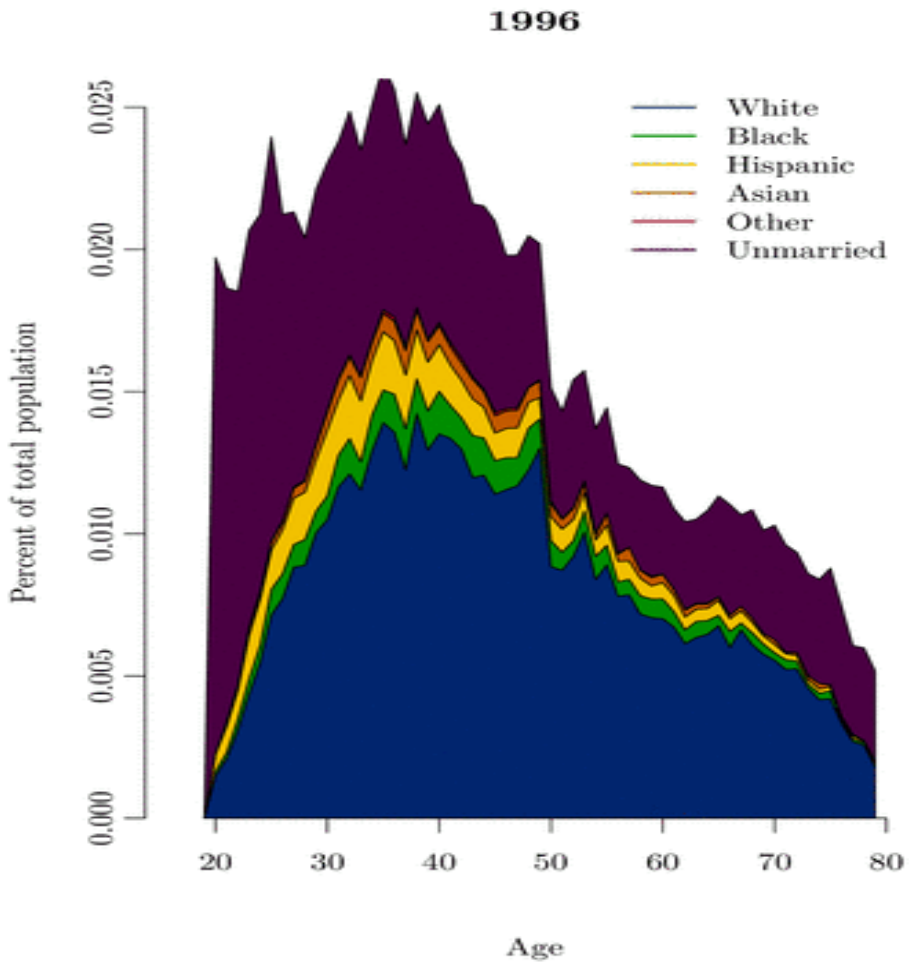


Microsimulation

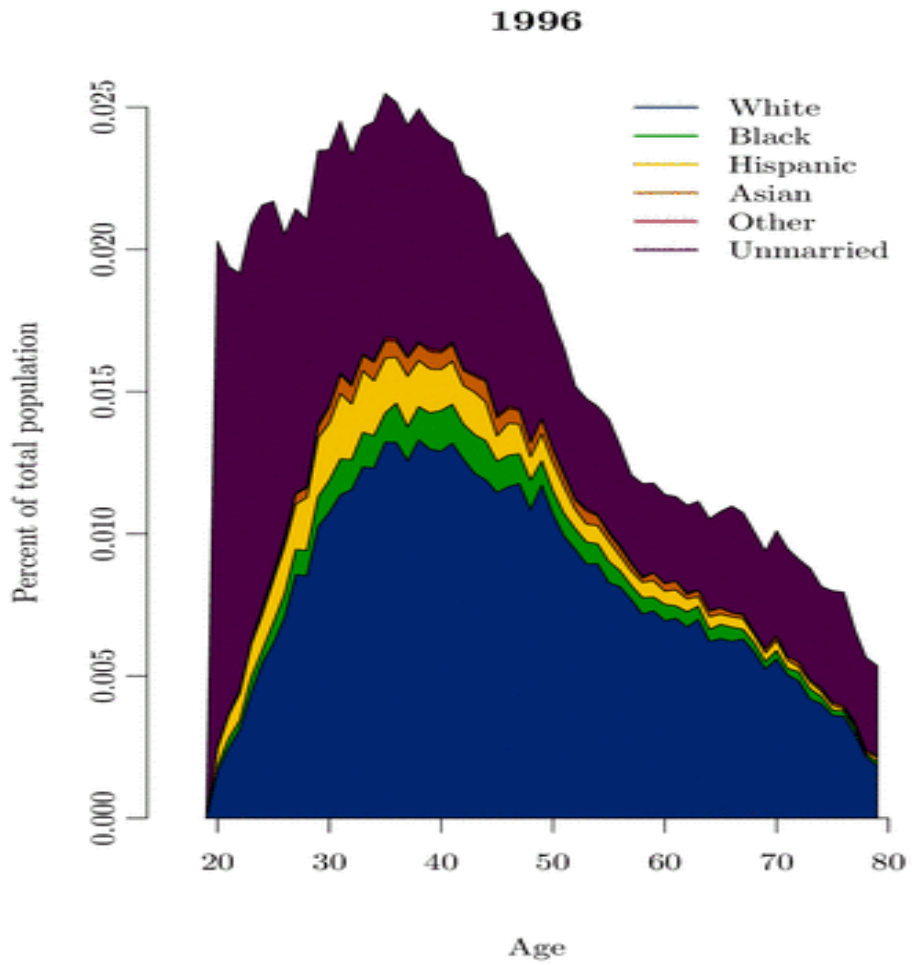


Marriage (1996 – 2070)

Census Data

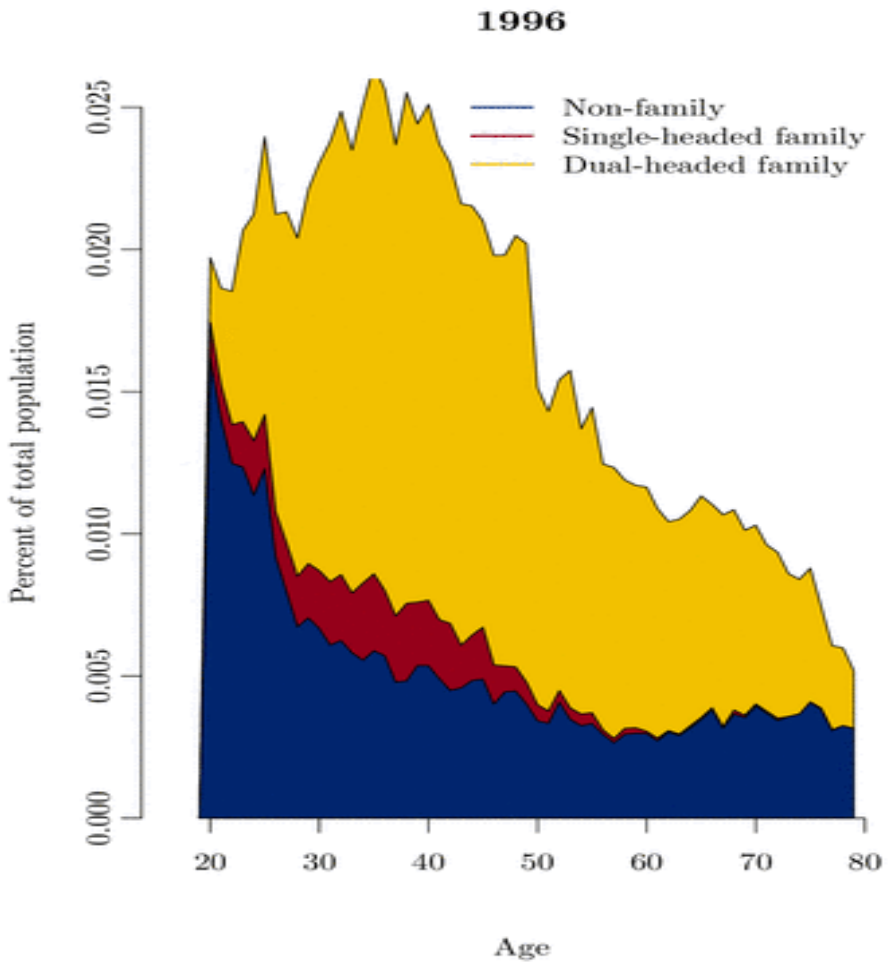


Microsimulation

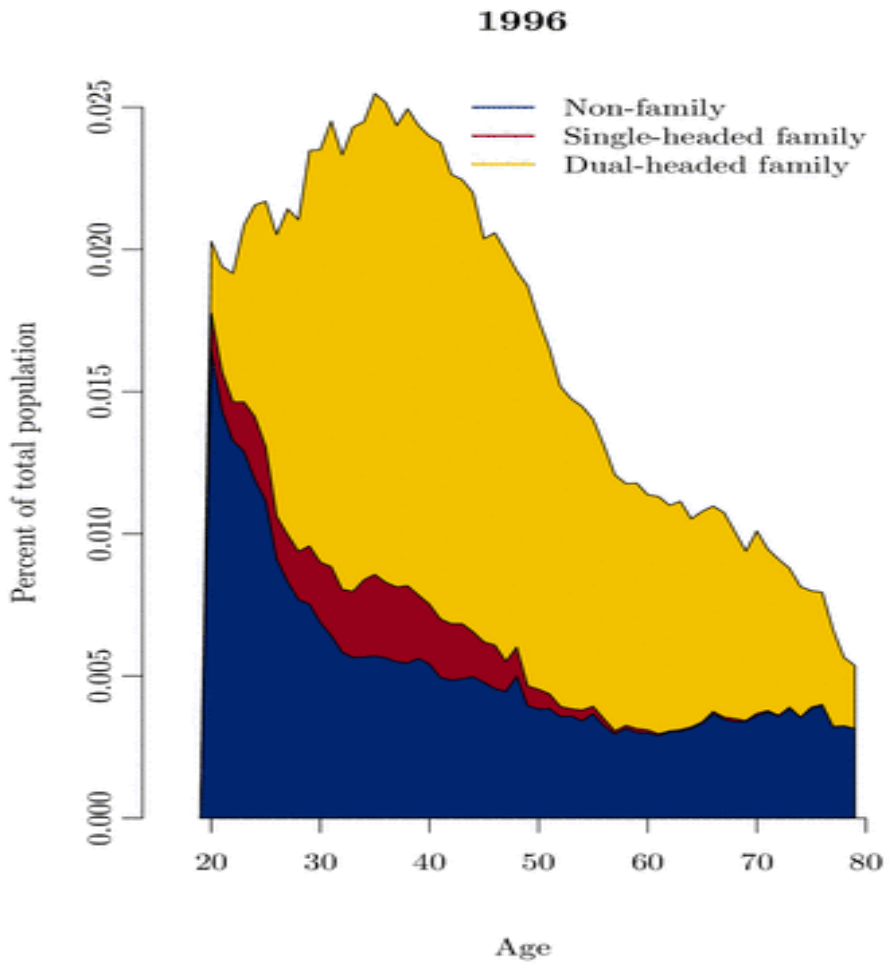


Family Composition (1996 – 2070)

Census Data



Microsimulation



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Briefly discussed next (and Appendix)

Labor Force: FTE Weeks worked

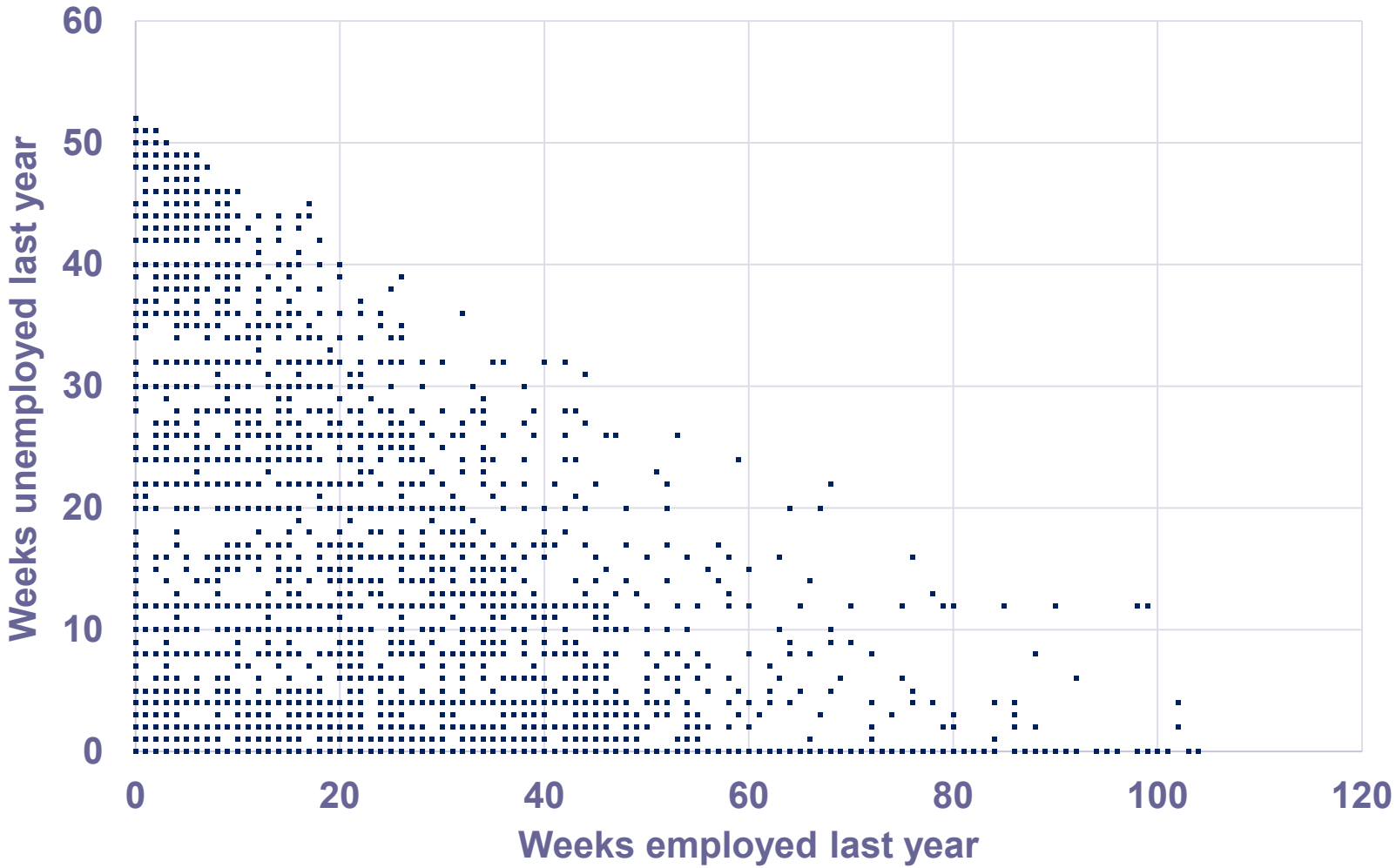
A. Determine FTE weeks employed (census micro-data; up to 104 per year)

- Initial FTE weeks regression: $\boldsymbol{\omega_t} = \boldsymbol{f}(age_t, gender, ethnicity, \dots)$
  - If $\boldsymbol{\omega_t} = \boldsymbol{0}$, set $\boldsymbol{ec_t} = \boldsymbol{0}$ … employment class = “not working”
- $\boldsymbol{ec}$ Transition (0/1/2/3): $\boldsymbol{ec_t} = \boldsymbol{Tec}(\boldsymbol{ec_{t-1}}|age_t, gender, ethnicity, \dots)$
- Set FTE weeks: $\boldsymbol{w_k} = \boldsymbol{w}(\boldsymbol{w_{k-1}}, \boldsymbol{ec_t}|age_t, gender, ethnicity, \dots), \boldsymbol{k} > \boldsymbol{t}$

B. Regardless of the outcome for A, determine annual weeks unemployed

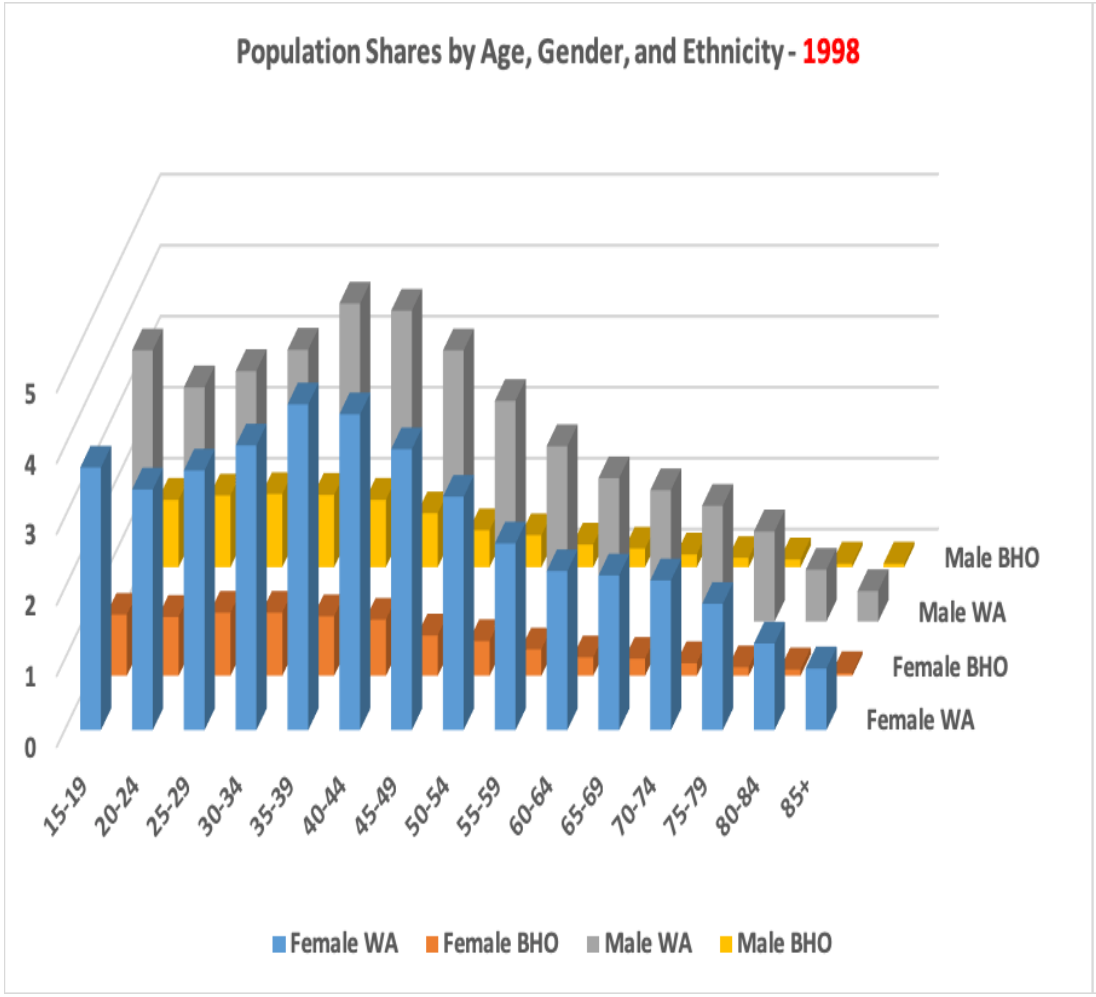
- $\boldsymbol{u_t} = \boldsymbol{u}(age_t, gender, ethnicity, \dots)$ = calendar weeks looking for work adjusted for cap on total weeks

Annual Work-weeks and Looking-for-Work weeks

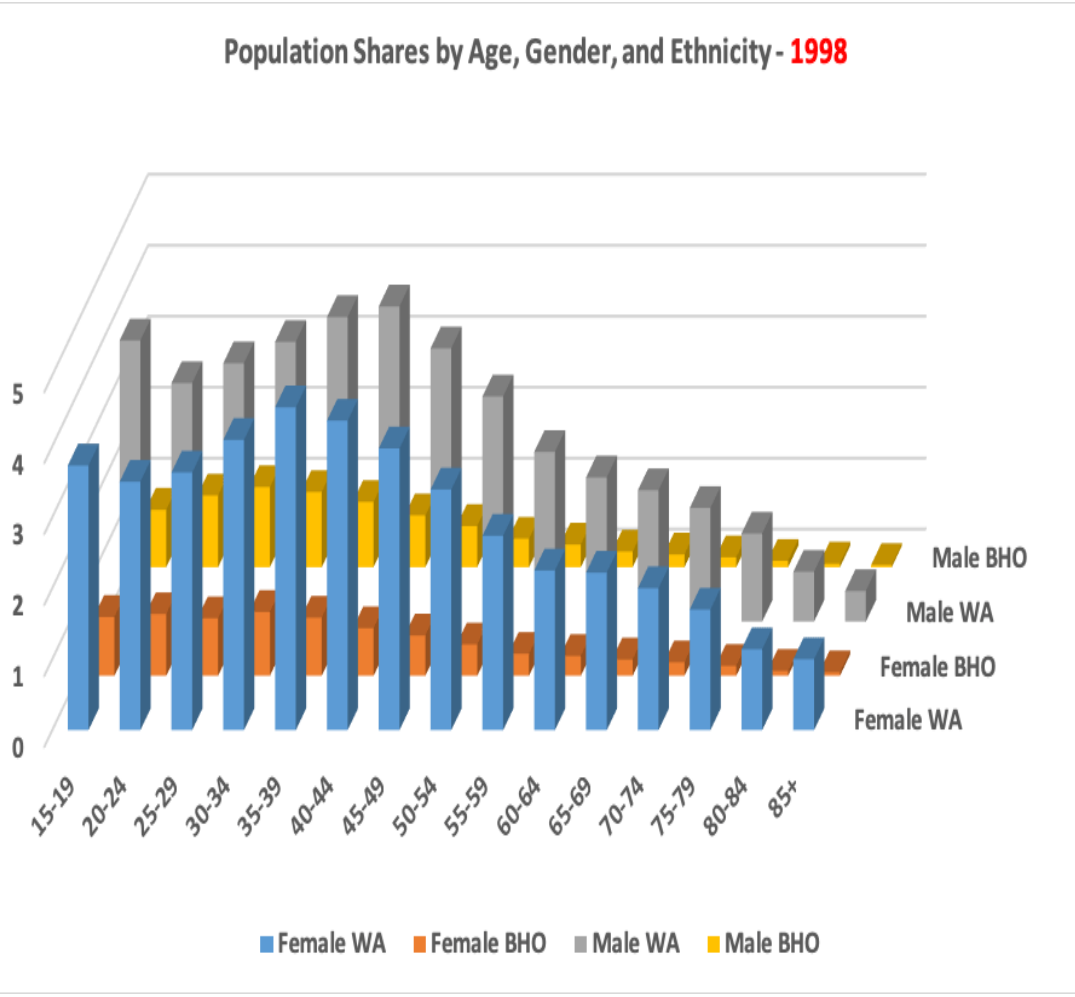


Labor Force Profiles

Census Data

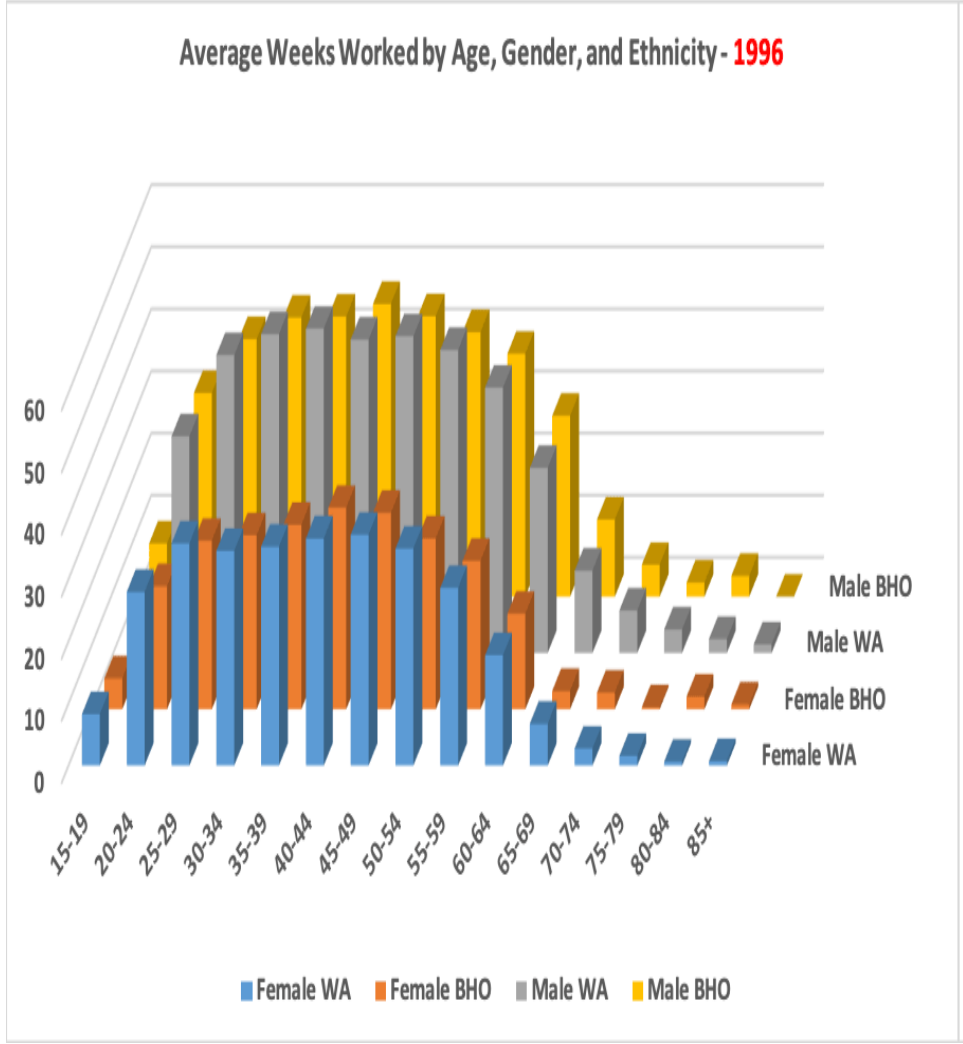


Microsimulation

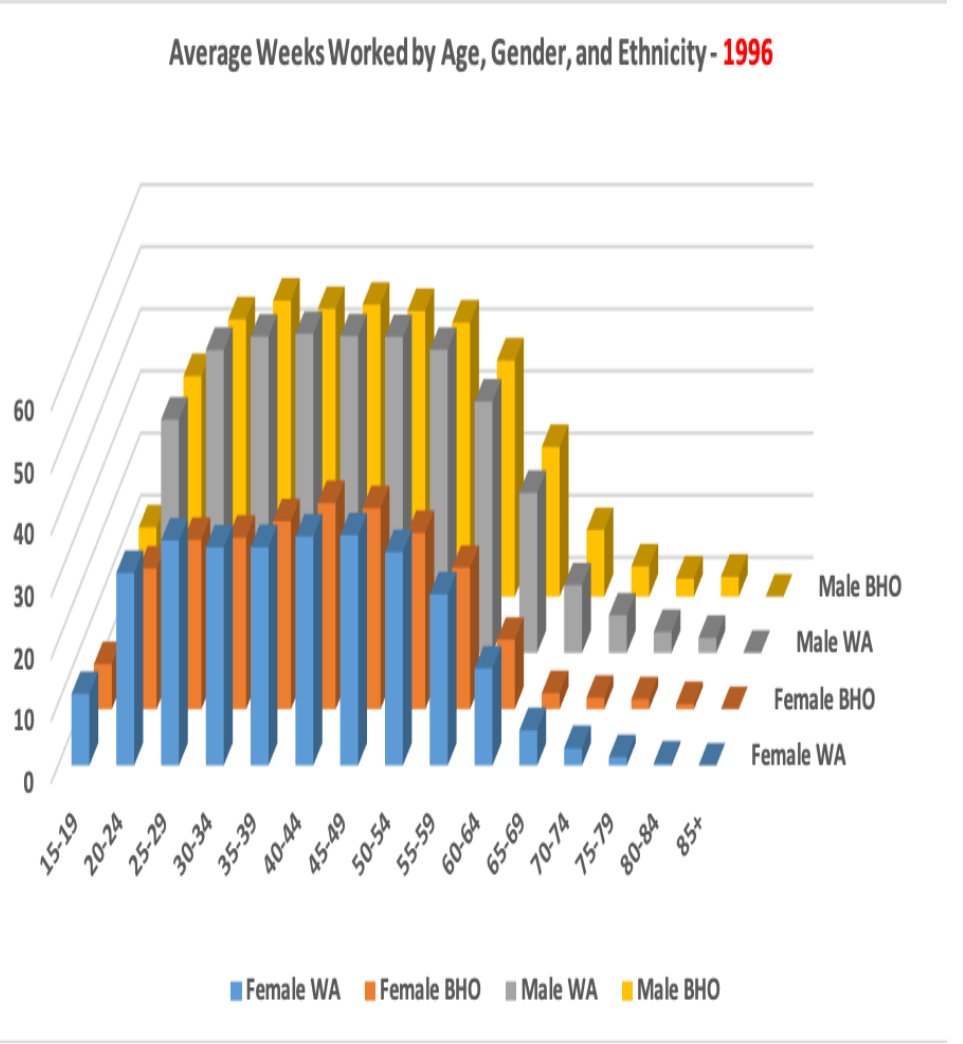


Labor Force Profiles

Census Data



Microsimulation



Aggregation: Capital, Efficiency-Adjusted Labor Services GDP, Labor Share and the Wage Base

Cobb-Douglas production function framework: Nominal GDP

$$Y_t = P_t A_t K_t^{\alpha} L_t^{1-\alpha}$$

Y_t = Total output
P_t = Price level
A_t = Multifactor productivity
K_t = Capital services
L_t = Efficiency adjusted labor services
α = Output elasticity of capital

See Appendix and PWBM website for estimation process

Labor Earnings

All worker characteristics determine efficiency at work per period (year)

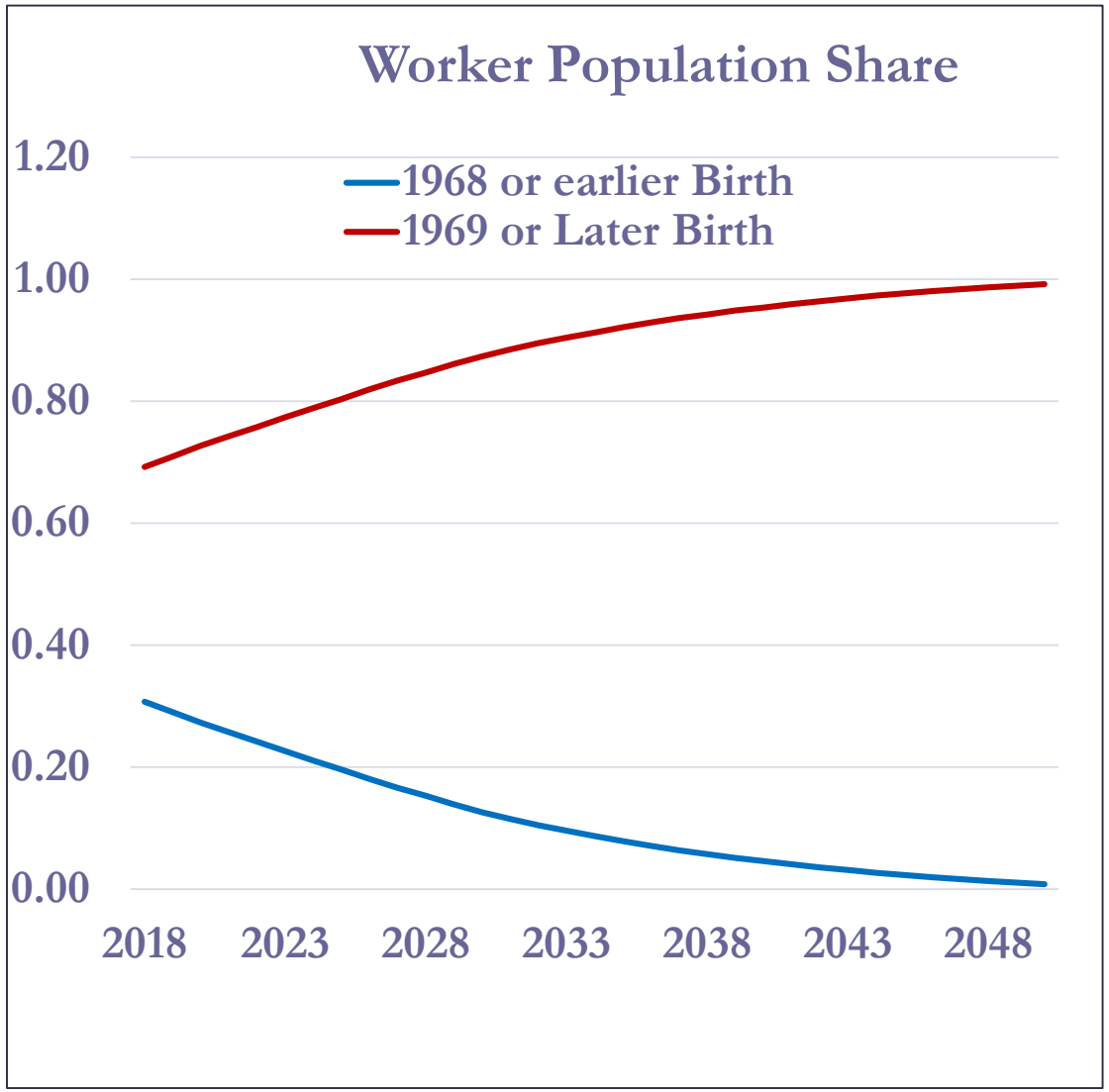
- **Age, gender, ethnicity, marital status, family size, FTE weeks/year, Unemployment weeks/year, health impairment, primary/secondary earner status, education years, birth-year, immigrant status, legal status, years since immigrated…with interactions**

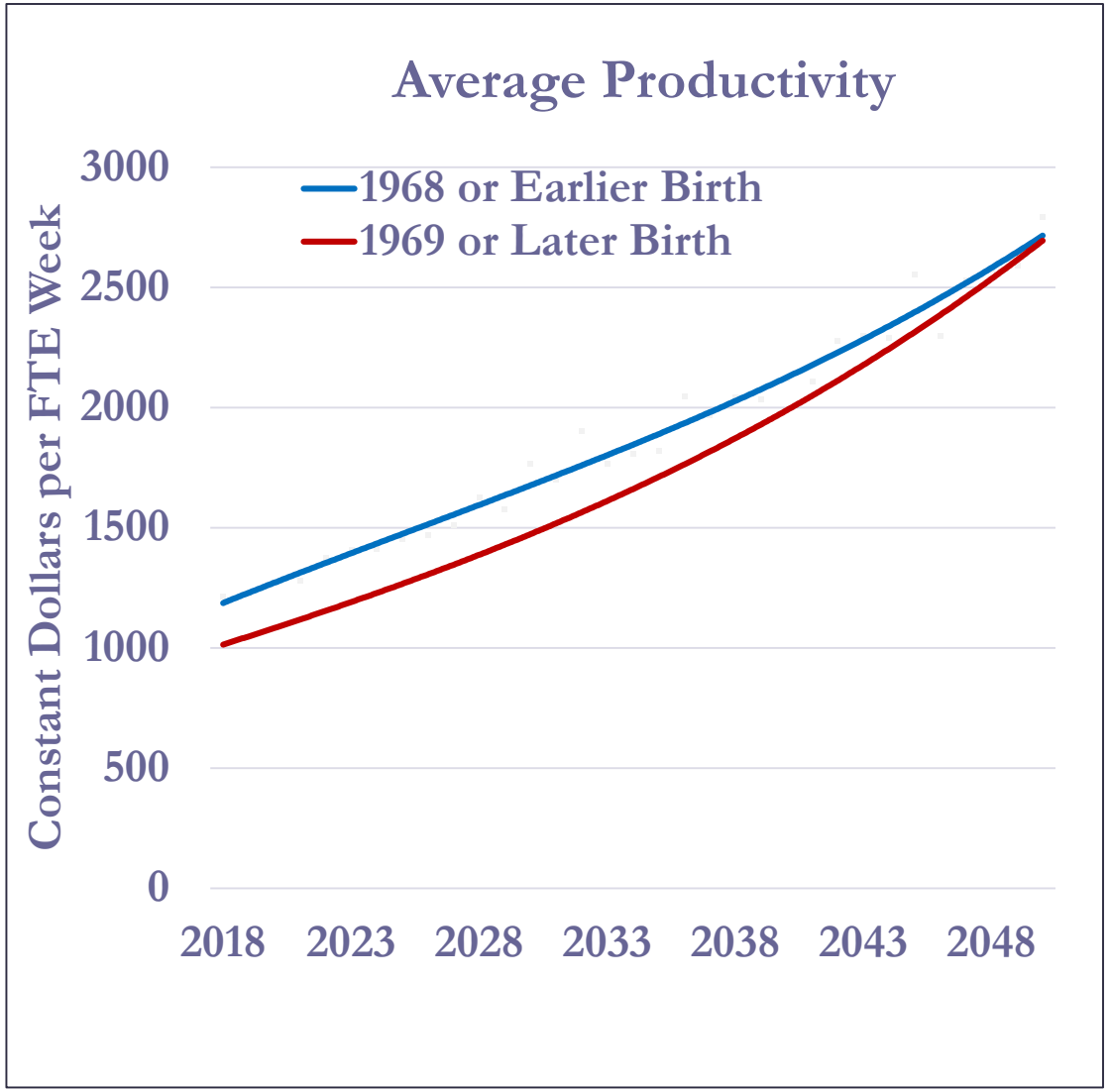
AR(1) regression on pooled cross-year data (see Appendix)

- **Isolates contribution of each worker attribute to productivity in the workplace**
- **Regression parameters can be applied to historical attributes and aggregated → GDP**
- **Can also be applied to projected worker attributes to project productivity according to distributions of worker attributes in future years**

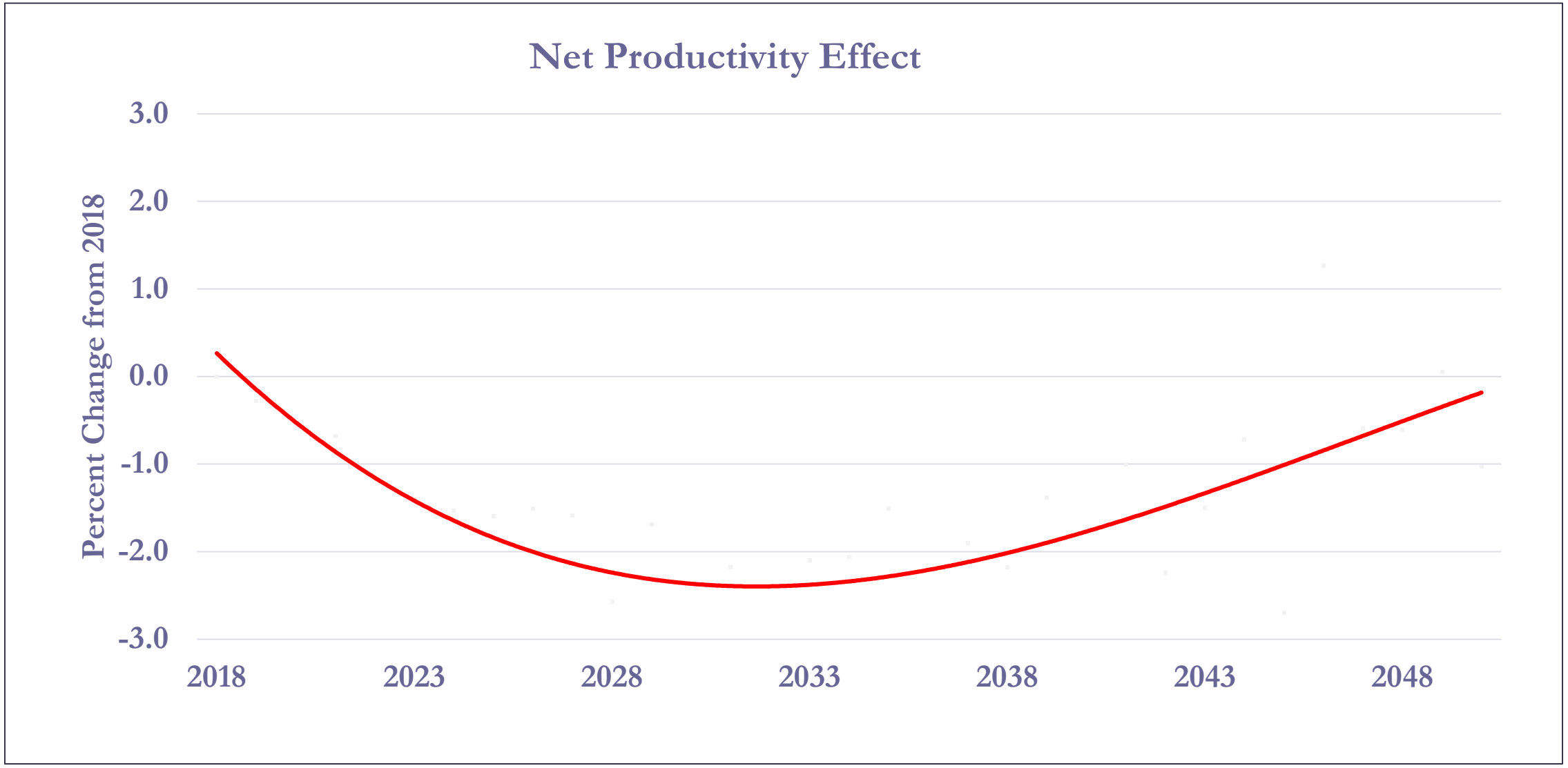
Productivity changes from compositional effects

Population Aging – Worker Population Split by 1968 Birth-year - 1

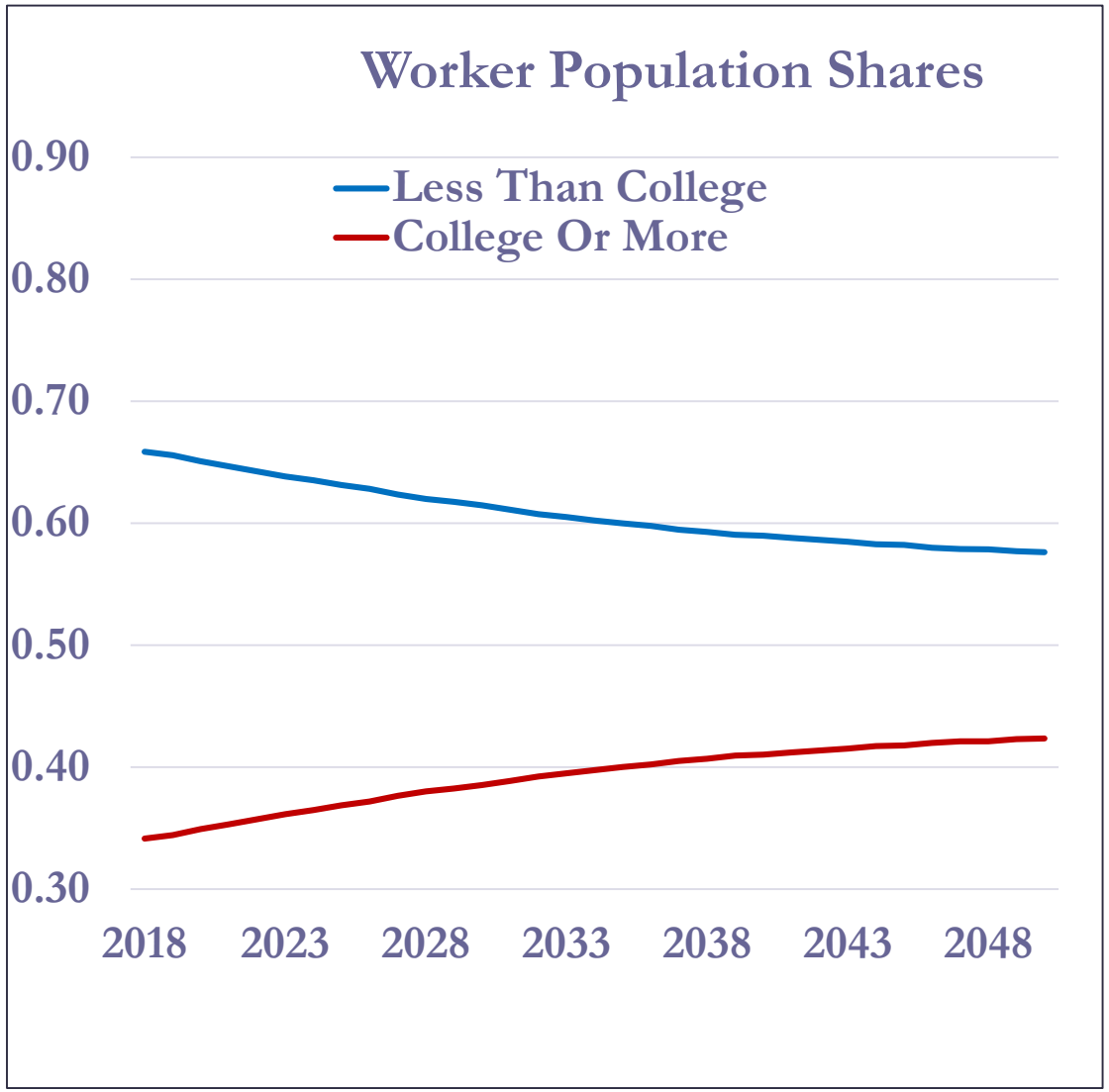


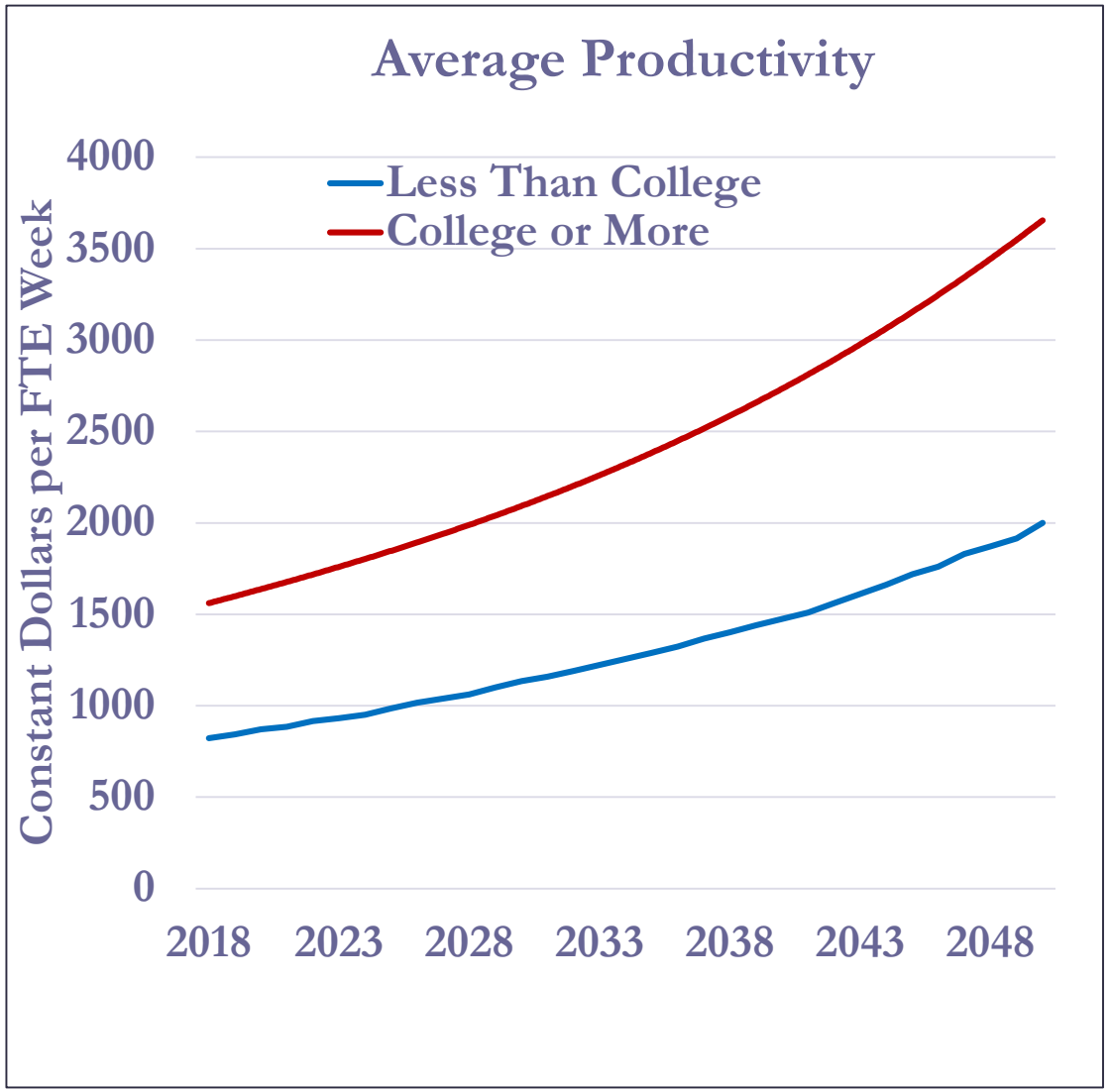


Population Aging – Worker Population Split by 1968 Birth-year - 2

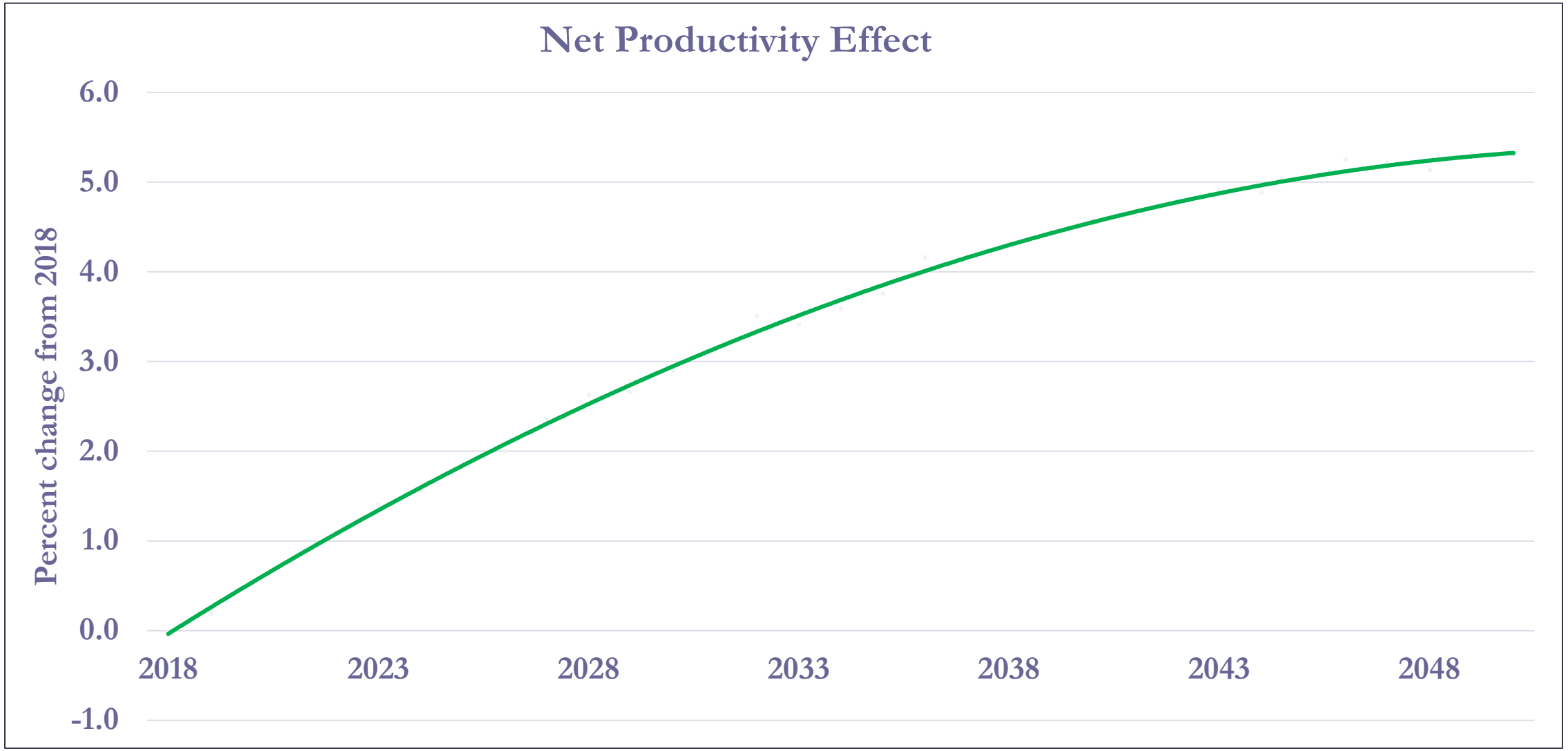


Worker Population by College or More - 1

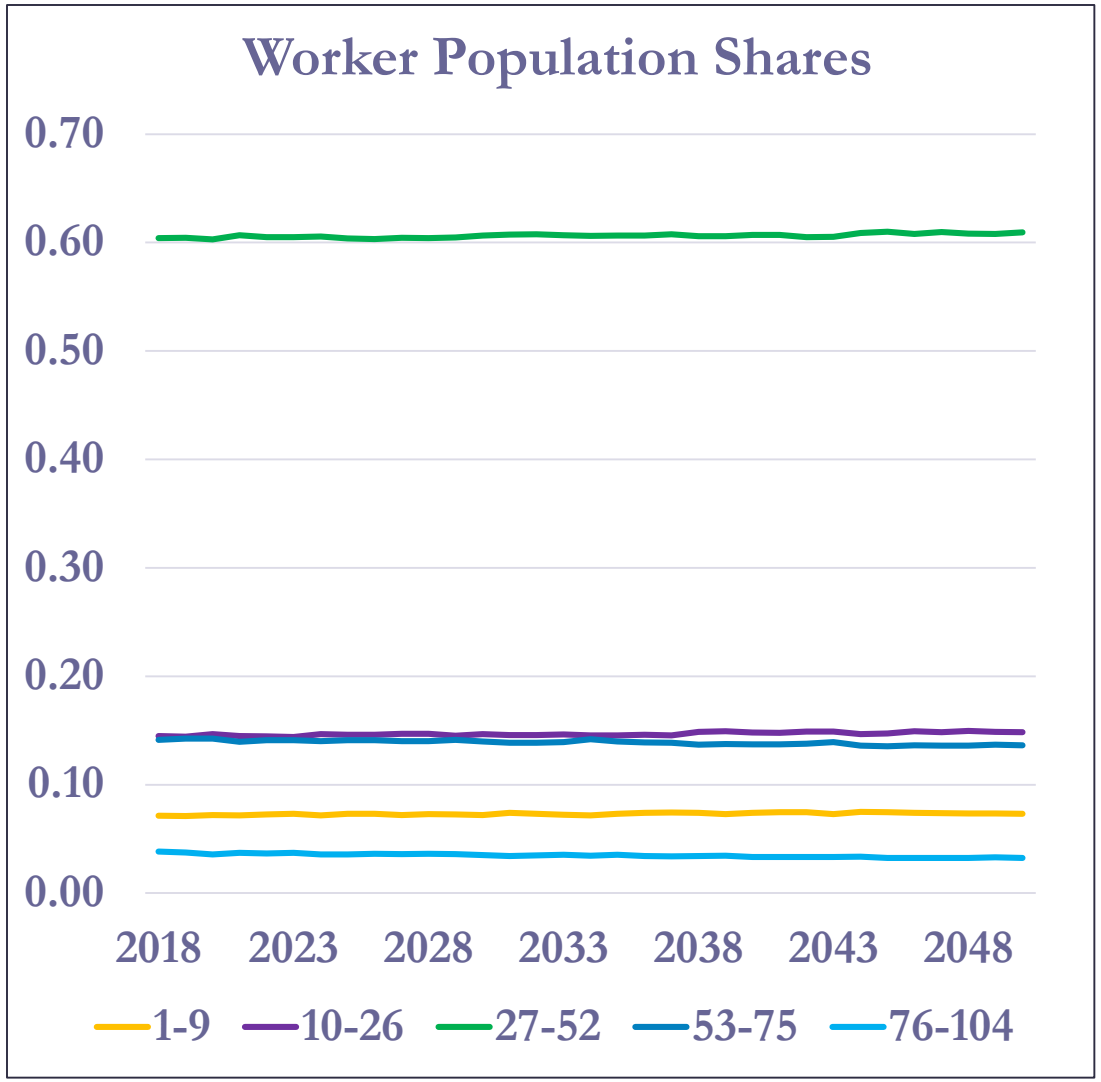


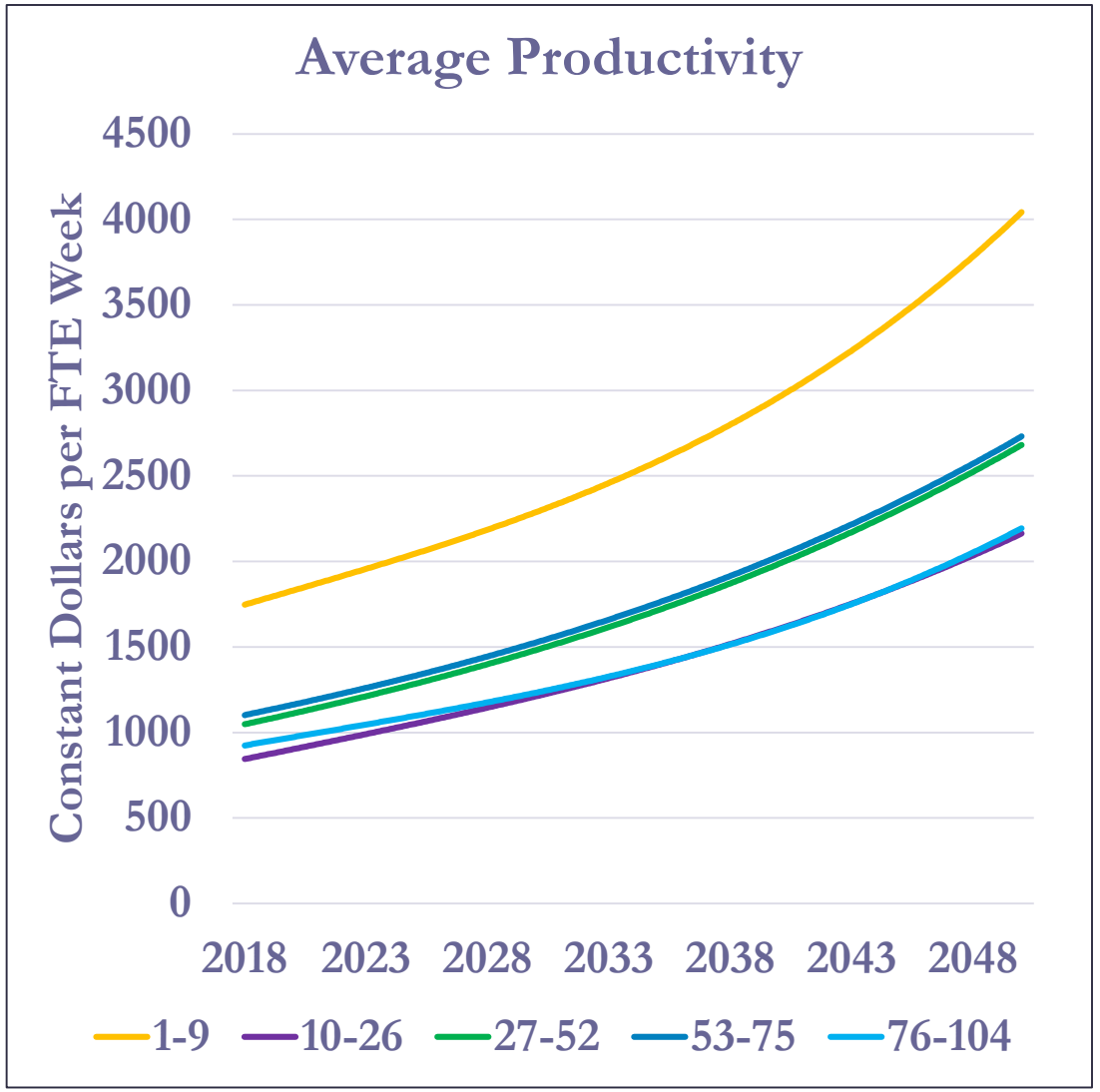


Worker Population by College or More - 2

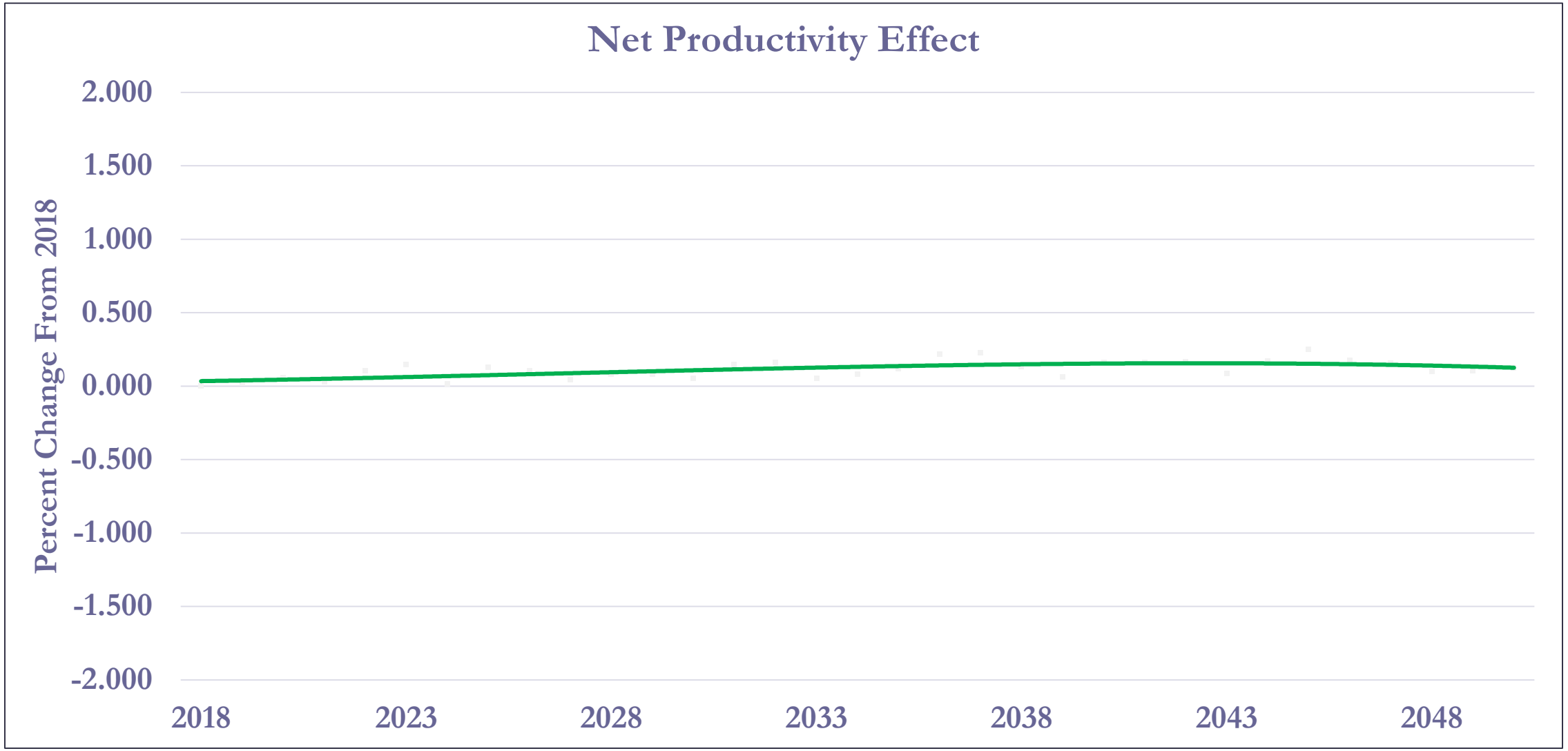


Worker Population by Annual Weeks Worked - 1

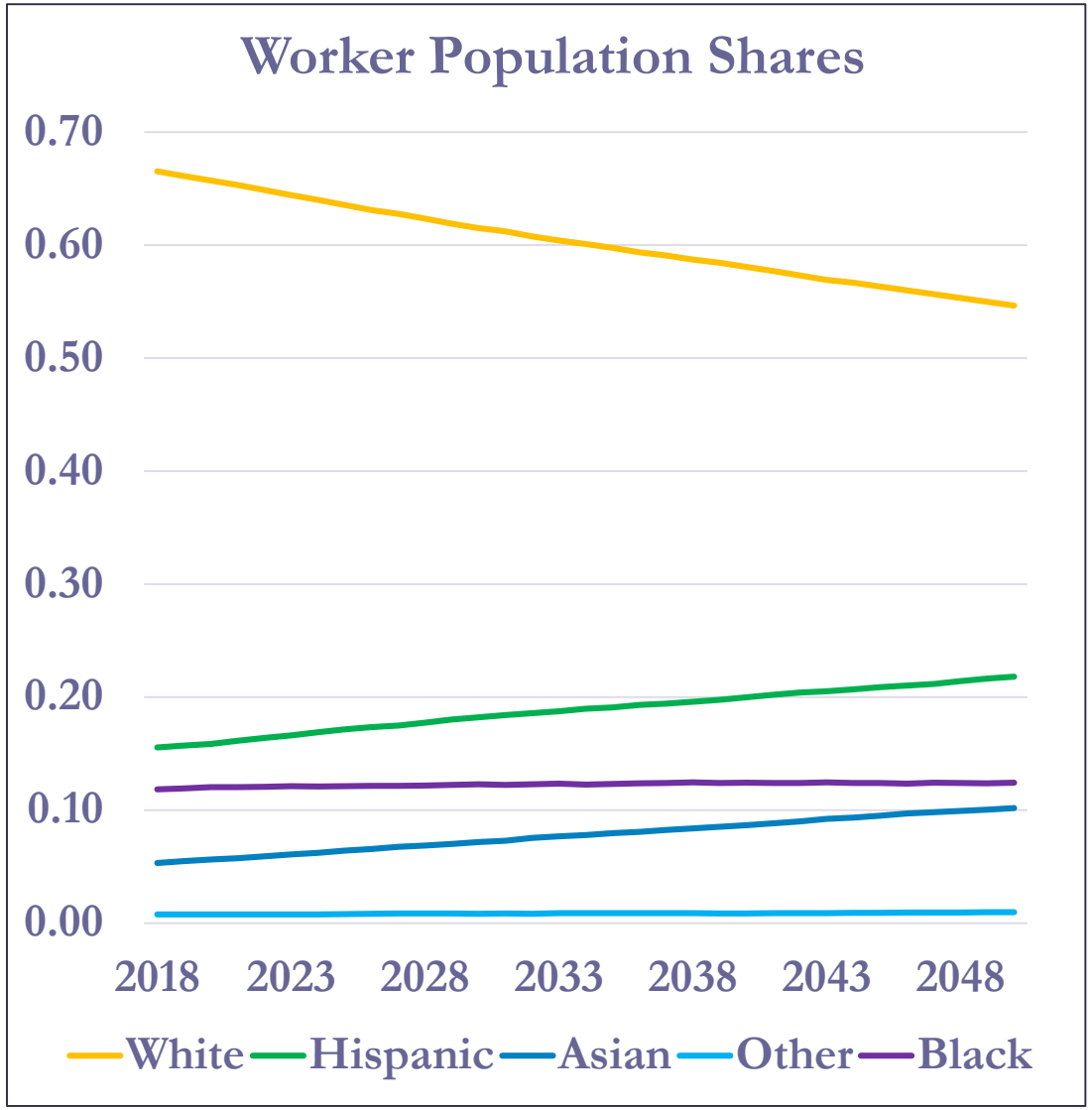


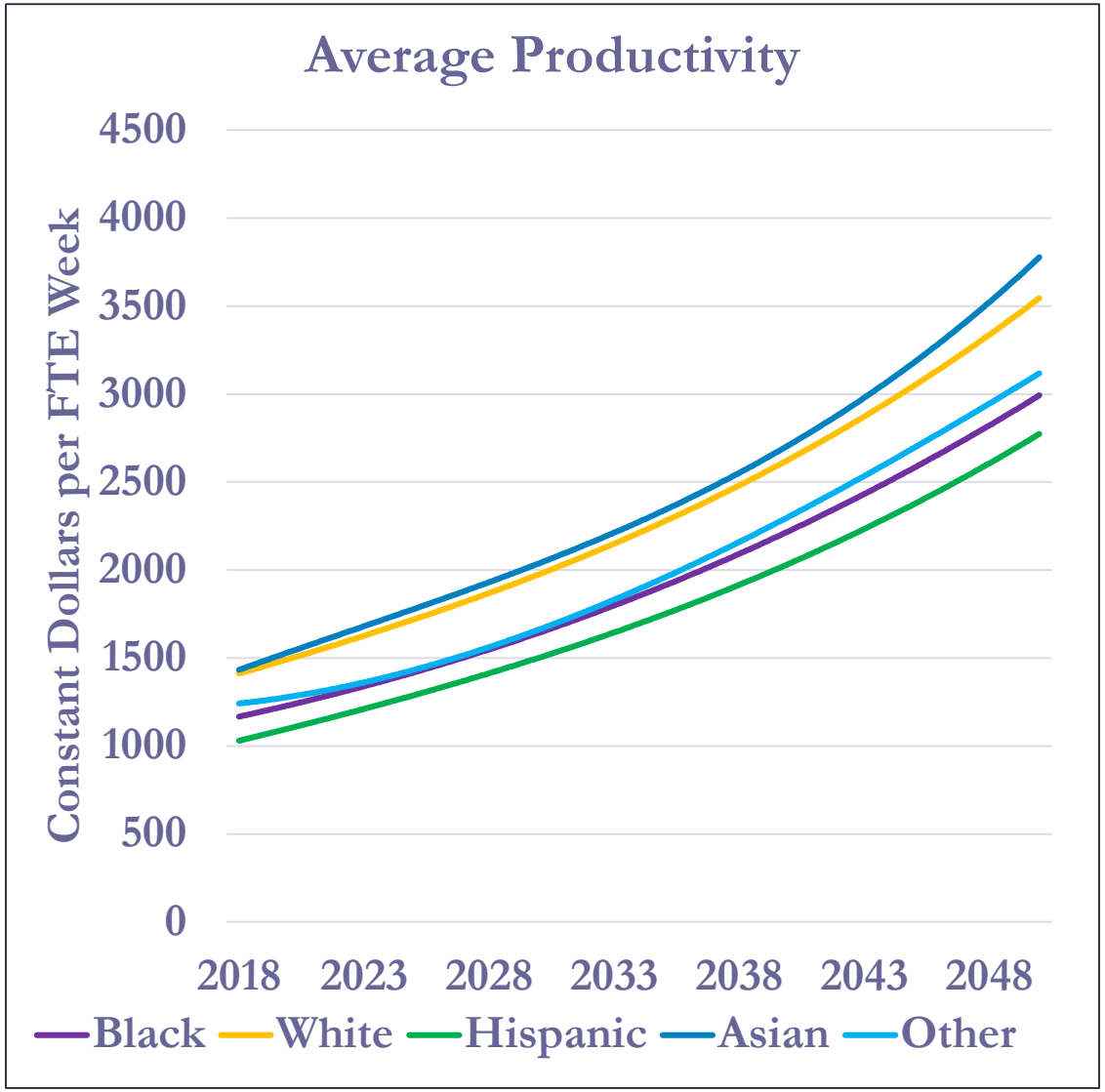


Worker Population by Annual Weeks Worked - 2

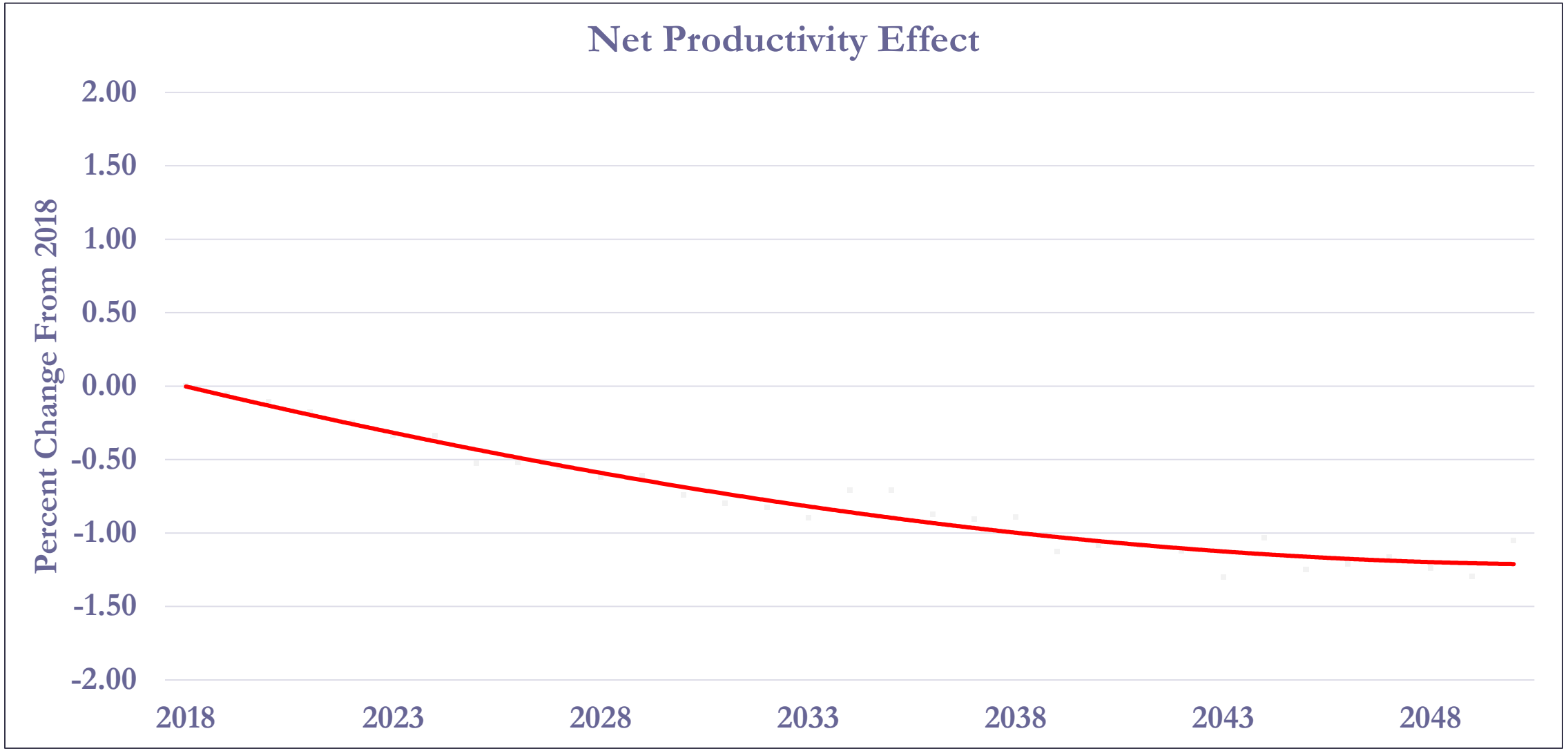


Worker Population by Ethnicity - 1

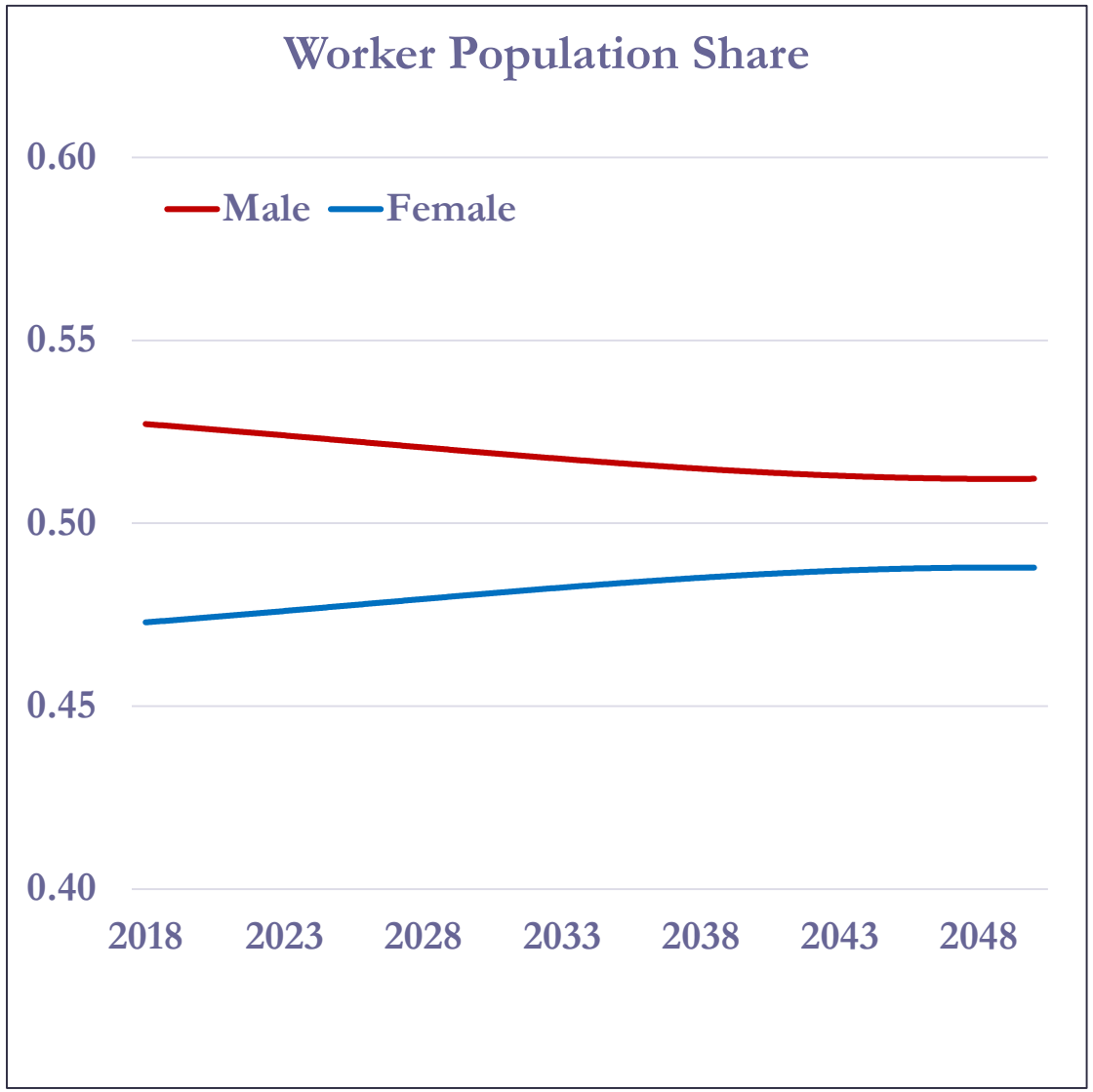


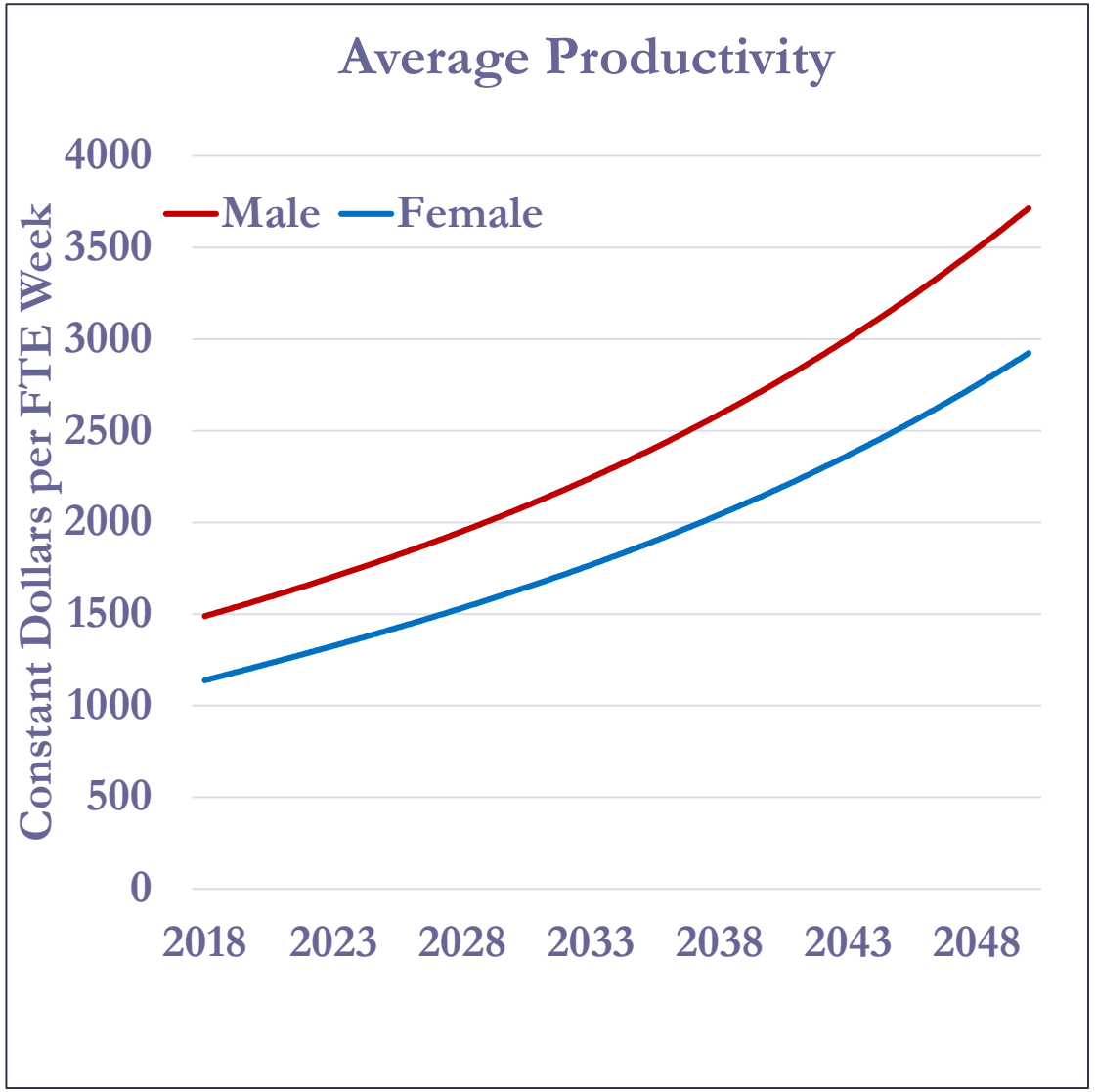


Worker Population by Ethnicity - 2

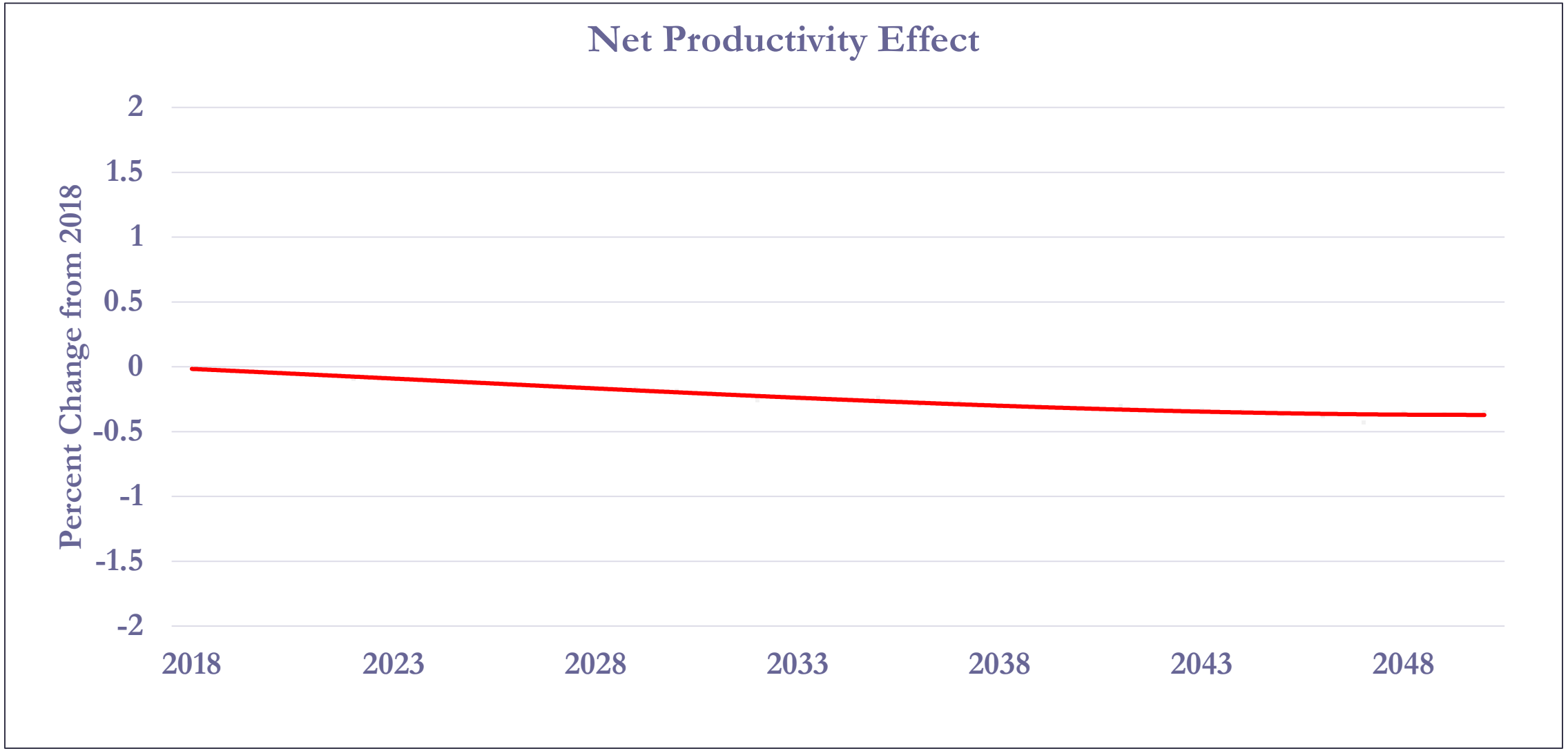


Worker Population by Gender - 1

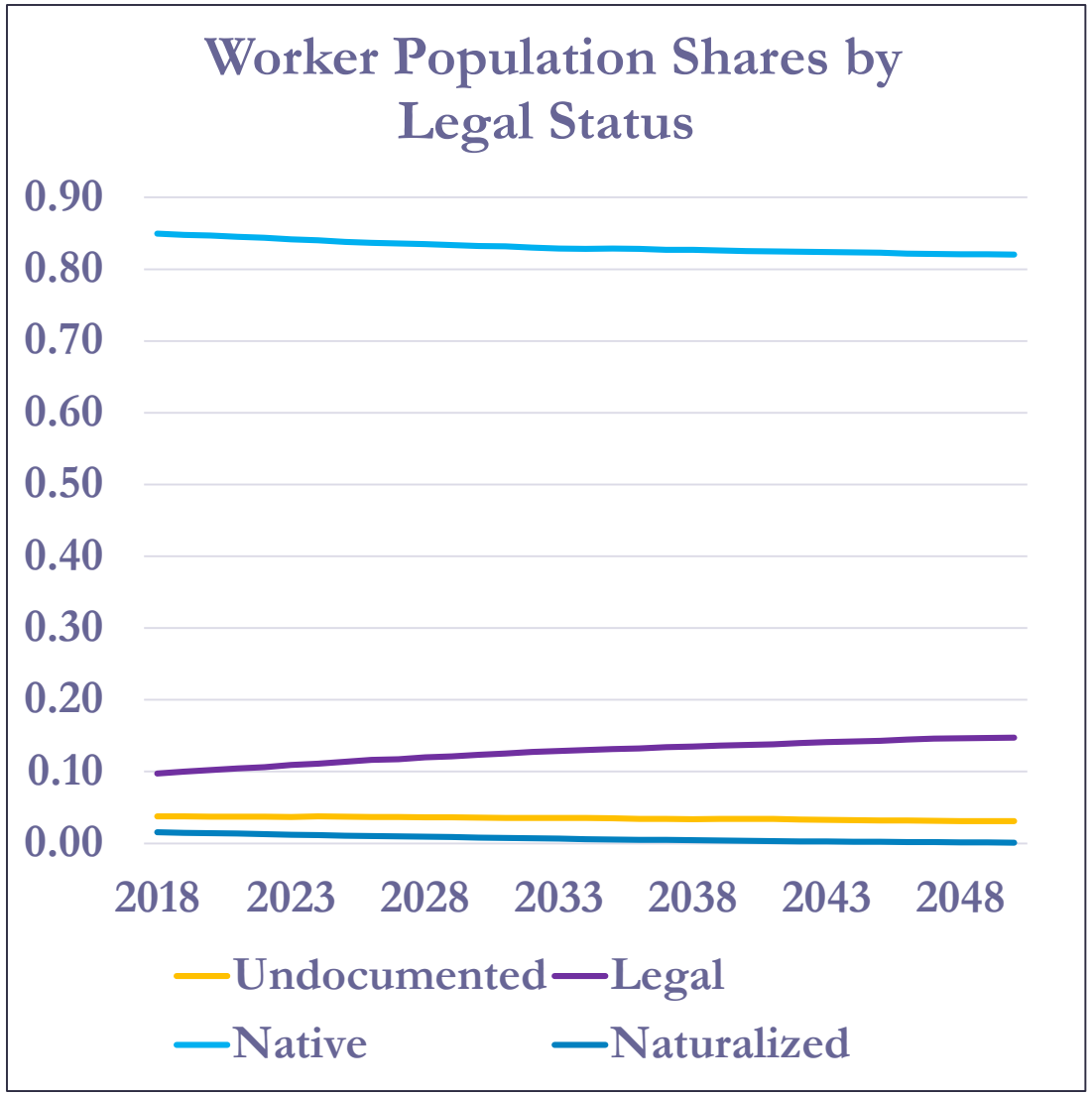


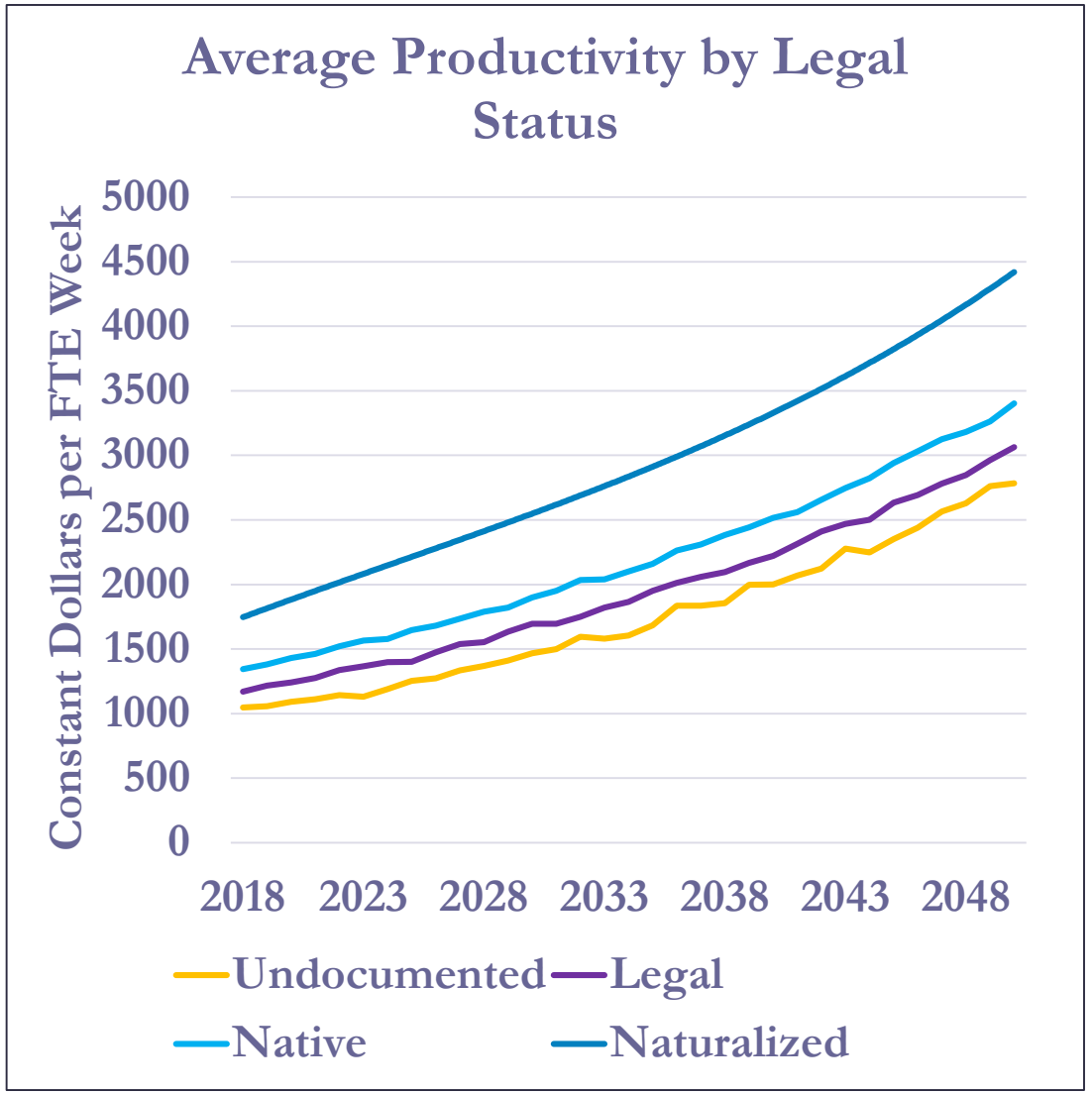


Worker Population by Gender - 2

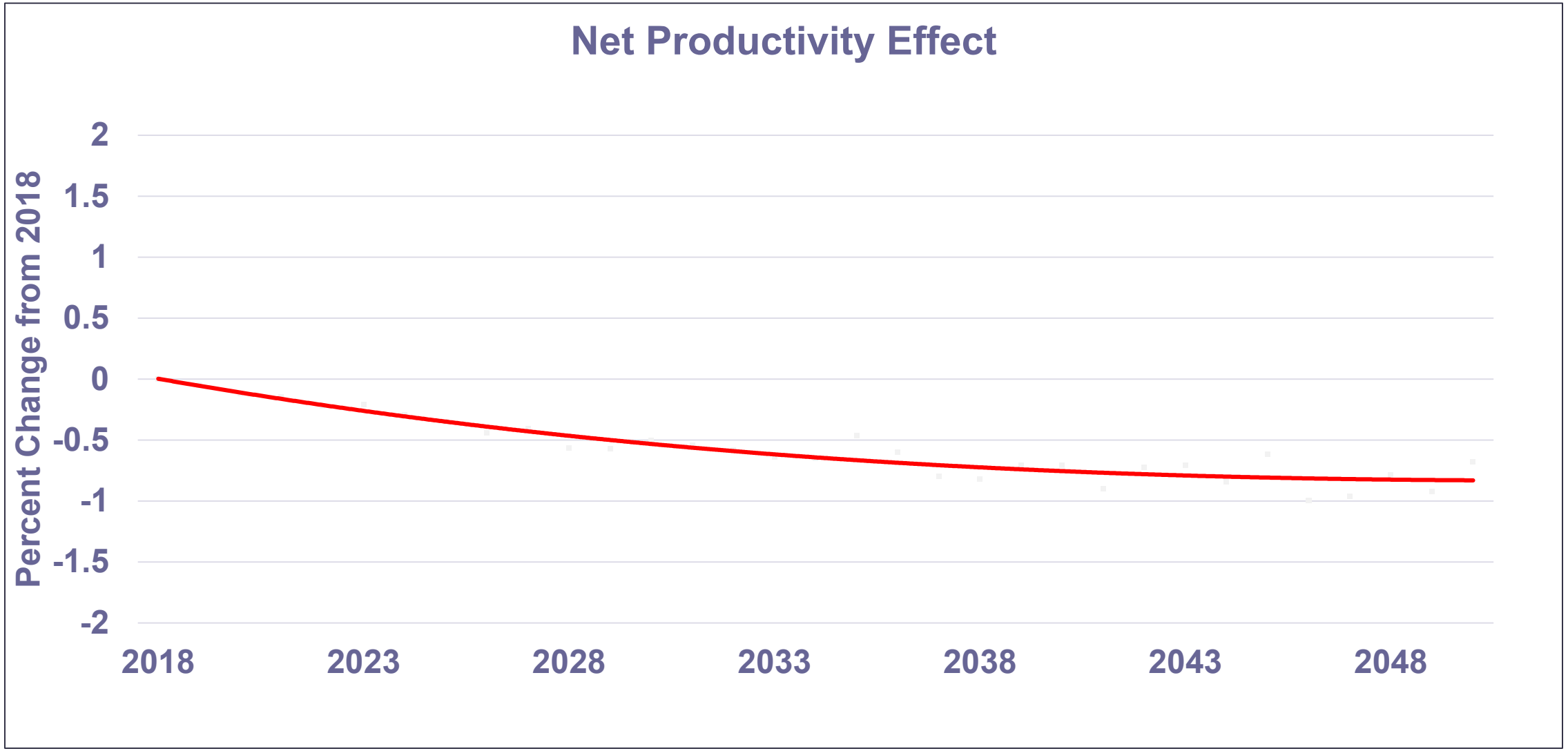


Worker Population by Legal Status - 1





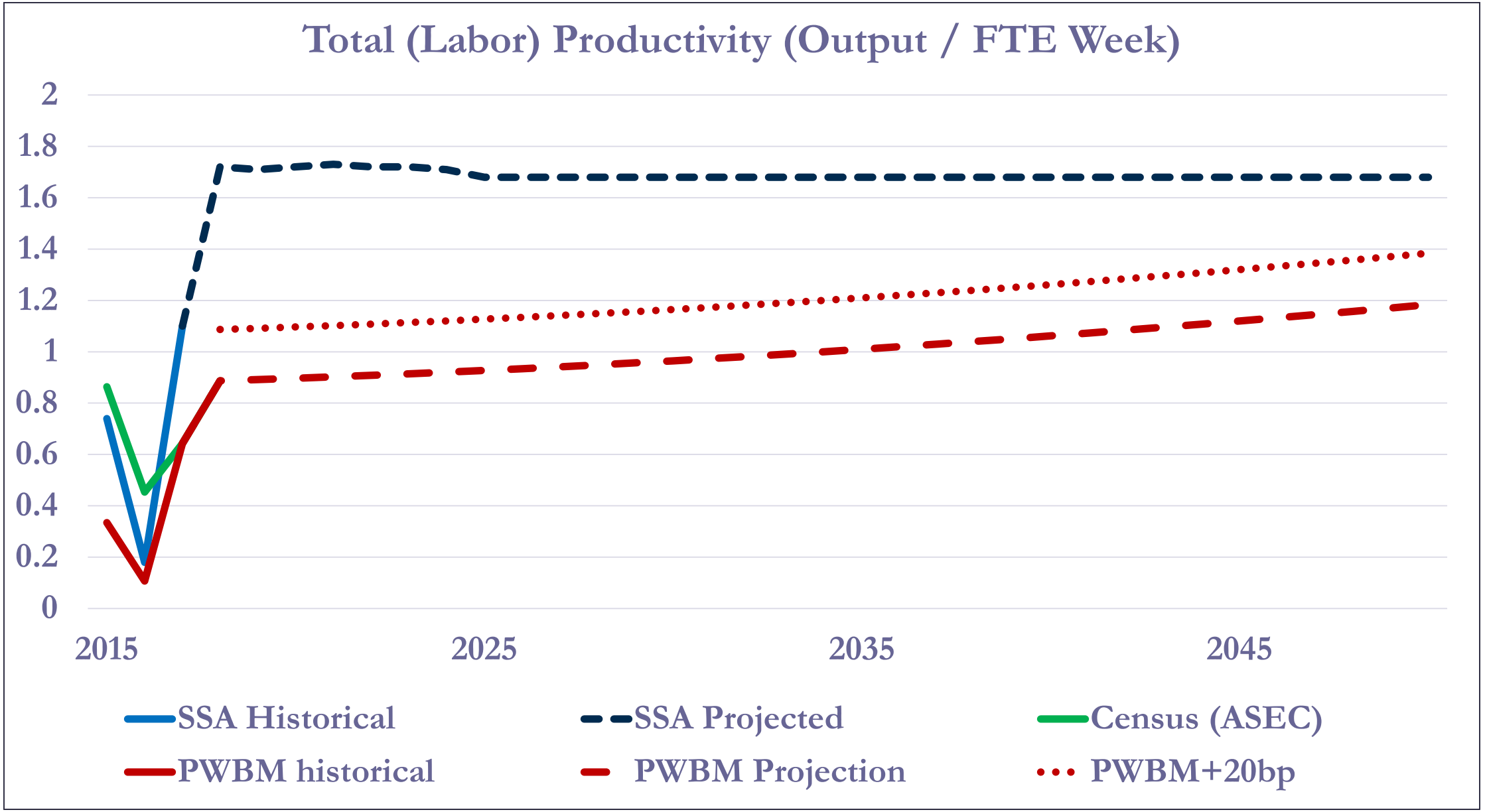
Worker Population by Legal Status - 2



Labor Productivity: Historical Averages

Source	Labor productivity average 1997-2017
Census ASEC (data)	1.46
PWBM (back test)	1.40
SSA (data)	1.64

Main difference between Census (data) and SSA (data) due to differences in construction of L term in productivity (Y/L). PWBM focuses on Census construction since we focus on a broader set of fiscal policies.


Total (Labor) Productivity (Output / FTE Week)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
2015
2025
2035
2045
SSA Historical
SSA Projected
Census (ASEC)
PWBM historical
PWBM Projection
PWBM+20bp

Individuals and Families: General Simulation Sequence

0	SIM Start	Initial population from ASEC with all attributes as of Dec. 31, 1996 aged 0-120
1	Aging	Add 1 year to each person's age: Age 0:119 --> ages 1:120; $A_t = A_{t-1} + 1$
2	Family split-offs	Those who turn 18 split-off and form their own family units: $p_{split}(\|age18) = 1$
3	Fertility	Females aged 14:49 → new age-0 pop. $f_t = f(age_t, ethn, educ_t, mar_t, \#kids_t)$
4	Mortality	Death rates: $d_t = d(age_t, sex, ethn, educ_t, mar_t)$; $d(age120) = 1$
5	Education	Age 6+ advance education years: $p(\Delta e) = p(e_{t-1}\|age_t, ethn, gender)$
6	Disability	People 0+ transit in-out of work impairment status (not SSDI): $\delta_t = d(\delta_{t-1}\|age, sex, ethn)$
7	LFP and FTE work hours	People 18+ through FTE weeks employed (0-104): $\omega_t = \omega(\omega_{t-1}\|age_t, sex, ethn, \ldots many)$
8	Employment	Those not working may be Unemployed: $u_t = u(\omega_t = 0\|age_t, sex, ethn, \ldots many)$
9	Immigration	Immigrants aged 0:119 (all attributes): $I_t^S = I_t^S \text{x} P_t \{i: immig\ rate;\ S: legal/undoc; P: pop\}$
10	Divorce	Divorce: Immediate entry into marriage market: $m_t = m(m\|age, sex, ethn, educ_t)$
11	Marriage	Marriage market (age 18+): $v_t = v(s\|age_t, sex, ethn, educ_t)$
12	Calculators	Wages and S. E. income Capital assigner (calibrated to BLS estimates of capital services) Benefits calculators: Social Security (other transfers under development) Tax calculators (Individual Income, Payroll, and Corporate)
13	Dynamic	GE-OLG Model with heterogeneous agents – attributes calibrated from SIM

Now (briefly)

Microsimulation: Taxes and Benefits

Benefits calculator

- Very detailed rules (even the rounding / truncation rules)
- Validated using 20,000+ different individual & household types and compared against OACT FORTRAN. All must be within one penny.

Tax calculator can be run on multiple bases:

- "Static" (no elasticities)
- "Conventional"
  - Business entity type elasticity (when appropriate)
  - Income deferral elasticity (when appropriate)
- "Dynamic" (with OLG model)
  - Labor supply elasticity to net tax (PVB - PVT at margin)
  - Allows for GDP / tax base growth effects

Macro-Model Integration

Integrated Micro-simulation and Stochastic OLG model

Key inputs reflect the detailed demographic and policy heterogeneity from the microsimulation model

- Social Security, individual and business income, taxes, transfers, etc.

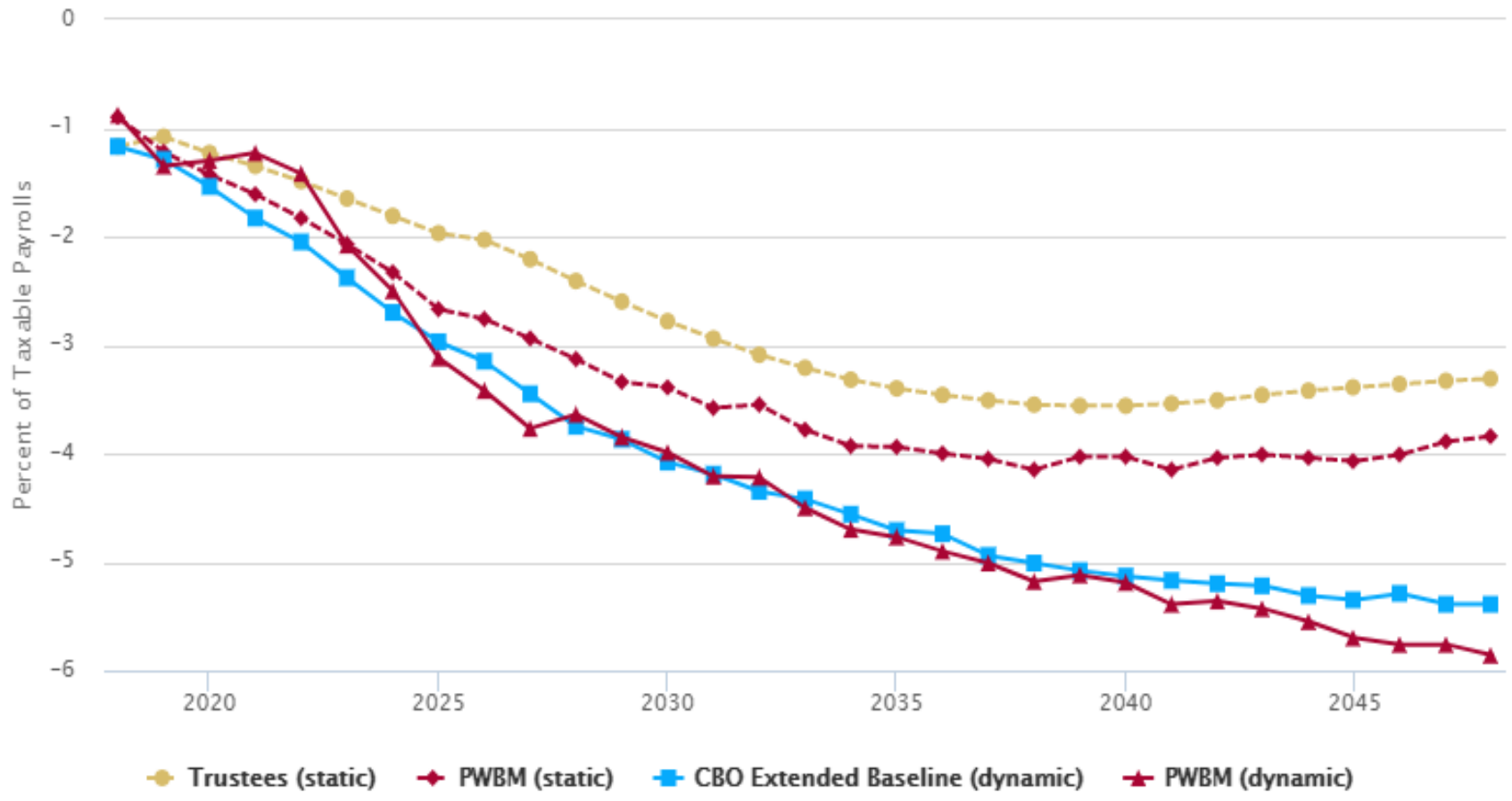
OLG model single firm, multi-agent Bewley-type model, with government

- Heterogeneity – age, productivity, wealth, immigrant status, earnings
- Government – taxes, transfers, “unproductive spending,” and debt

Baseline and policy runs: Equilibrium time paths of household and firm decision rules and macro time-series

Summary

Figure 4: Social Security (OASDI) Projected Annual non-Interest Balance Ratios (percent)



Appendix

Aggregation: Capital, Efficiency-Adjusted Labor Services → GDP, Labor Share and the Wage Base - 1

Cobb-Douglas production function framework: Nominal GDP

$$Y_t = P_t A_t K_t^{\alpha} L_t^{1-\alpha}$$

Y_t = Total output
P_t = Price level
A_t = Multifactor productivity
L_t = Capital services
L_t = Efficiency adjusted labor services
α = Output elasticity of capital

Rewrite: $lnY_t - lnP_t - lnA_t - \alpha lnK_t = (1-\alpha) lnL_t$

P, A, and $\bar{\alpha}$ are specified exogenously, while K and L are determined within the simulation…How?

Aggregation: Capital, Efficiency-Adjusted Labor Services → GDP, Labor Share and the Wage Base - 2

Nominal Compensation Share:

$$W_t \equiv (1-\alpha)Y_t = (1-\alpha)P_t A_t K_t^{\alpha} L_t^{1-\alpha}$$

Rewrite and adjust for prices, MFP, and capital deepening:

$$ln\, Z_t = ln\, W_t - lnP_t - lnA_t - \alpha lnK_t = \ln(1-\alpha) + (1-\alpha) lnL_t$$

Efficiency adjusted labor input is modeled as

$$L_t = \sum_i l_{it} = \sum_i e^{\sum_{j=1}^{m} \theta_j x_{jit}}$$

Cross-year stacked regression at worker level, annual freq.:

$$lnL_t = \frac{ln\, z_{it} - \ln(1-\alpha)}{(1-\alpha)} = f(x_{it};\theta) = \sum_{j=1}^{k} \theta_j x_{jit} + u_{it}$$

Aggregation: Capital, Efficiency-Adjusted Labor Services → GDP, Labor Share and the Wage Base - 3

- Regression executed on gross wages although labor share includes employee benefits
  - Wages are observed in micro-data, total compensation is not
  - Estimate and add non-wage compensation: social security employer taxes, pension and health insurance benefits (based on simulated coverages), and other compensation
    - Social Security employer taxes are easy!
    - Pension and health benefits assumed proportional to simulated wage; benchmarked to national totals
    - Nonwage benefits benchmarked to national total by adjusting other benefits
- Stacking observations from different years (1996-2016) - cross-year wages have been "placed on par" by removing the effect of inflation, MFP, and capital deepening
- Assume all workers hired on spot-market - no long-term implicit contracts that cause current compensation to diverge from current productivity → Observed wage = worker's current productivity

Aggregation

$$\tilde{L}_t = \sum_i \tilde{l}_{it} = \sum_i ln\tilde{z}_{it} = f(\tilde{x}_{it}; \hat{\theta}) = \underbrace{\sum_{j=1}^{k} \hat{\theta}_j \tilde{x}_{jit}}_{\text{core labor input}} + \underbrace{\hat{u}_{it}}_{\text{idiosyncratic shock}}$$

- Apply regression coefficients (^) to simulated (~) worker attributes →
  - Captures contribution of worker attributes to productivity – core labor input
- Shock term – segmented/tailored-bootstrap from distribution of regression errors
  - idiosyncratic shock - captures unexplained variability in worker productivity – assumed to be transitory

Aggregation: Issues

Observed work-choices and wage levels are not independent

- Wage regression on attributes w/o adjustment for selection

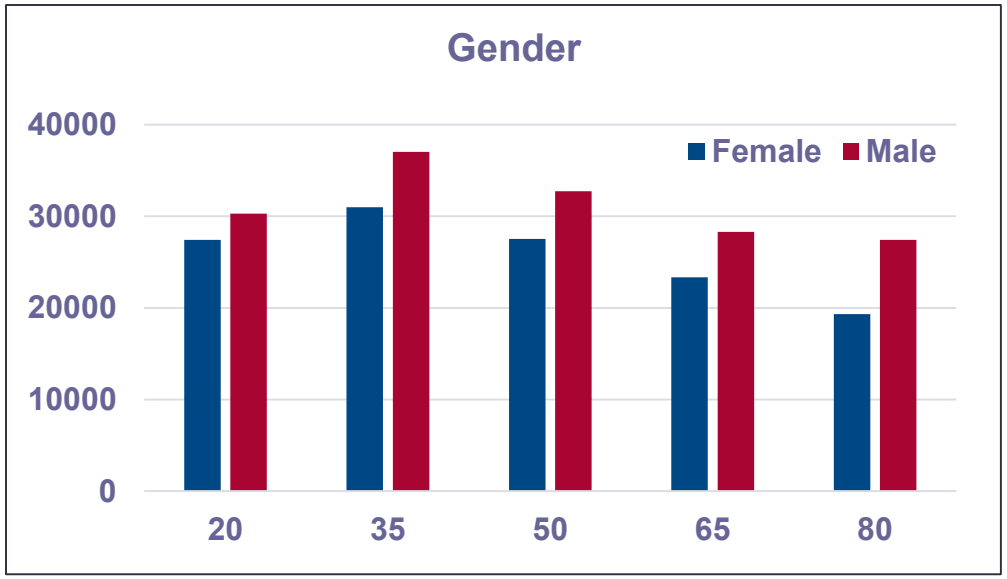
  → biased coefficient estimates

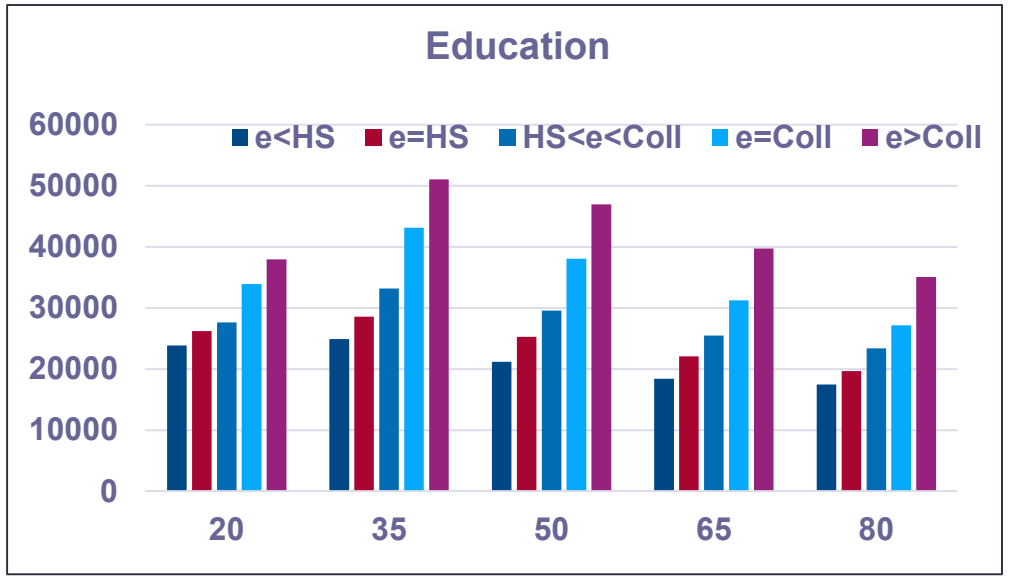
- Fixing the selection bias (adjust error term with Mills ratio) matters for estimating effects of worker attributes on potential productivity, that is, not conditioning on work choice
- But such a fix is not needed when simulating wages: coefficient estimates applied post selection of work choice estimated separately

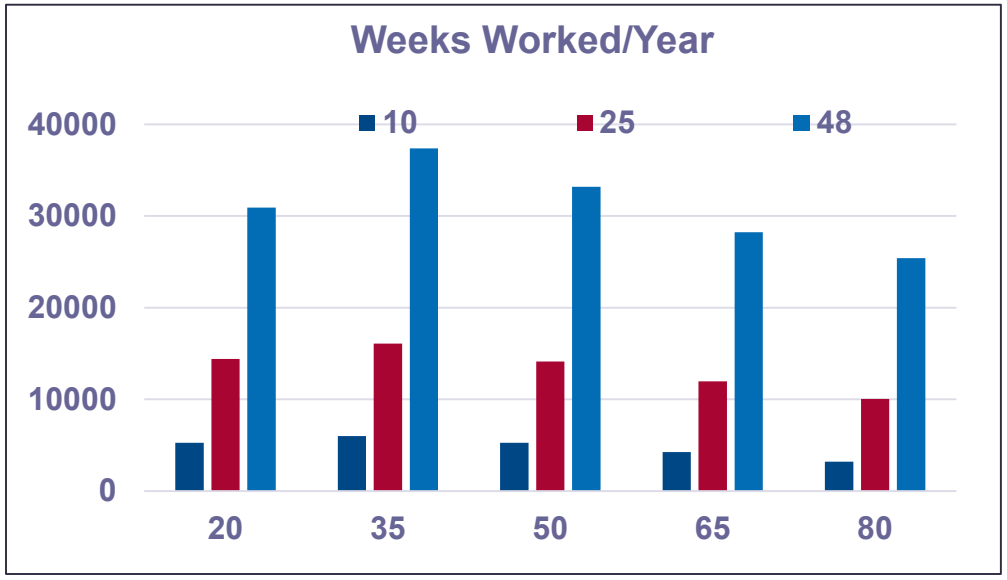
Current procedure

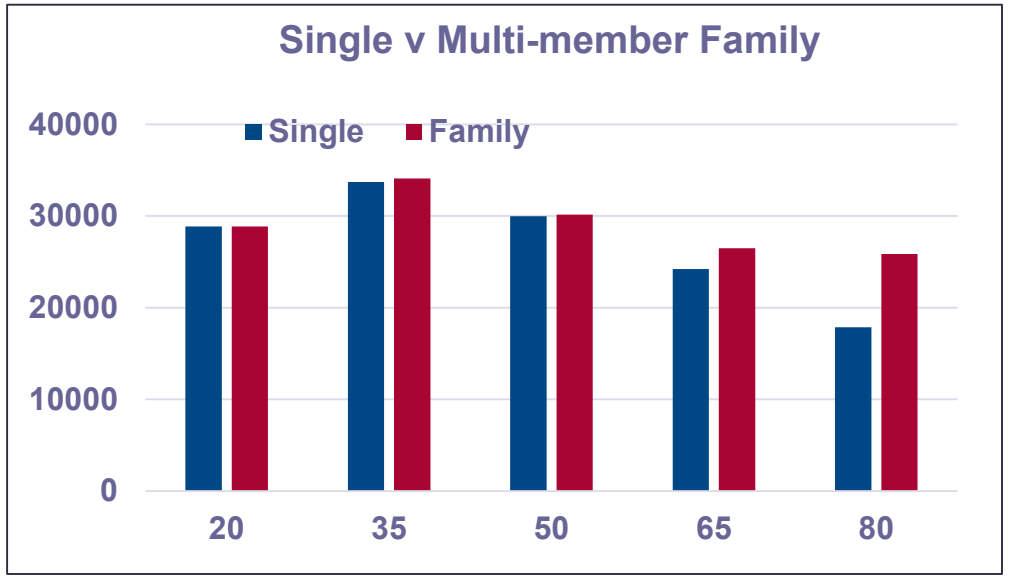
- Simulate work choice and FTE hours from micro-data (described above)
- Labor hours enter as an attribute in the wage-regression → core wages, lnL_t

Conditional Productivity Differences by Specific Attribute at Selected Ages

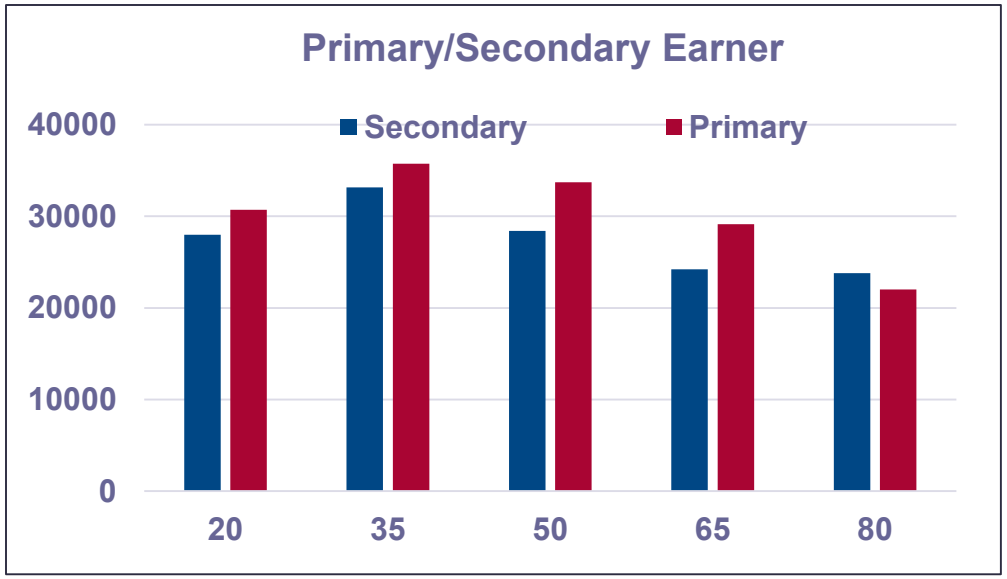


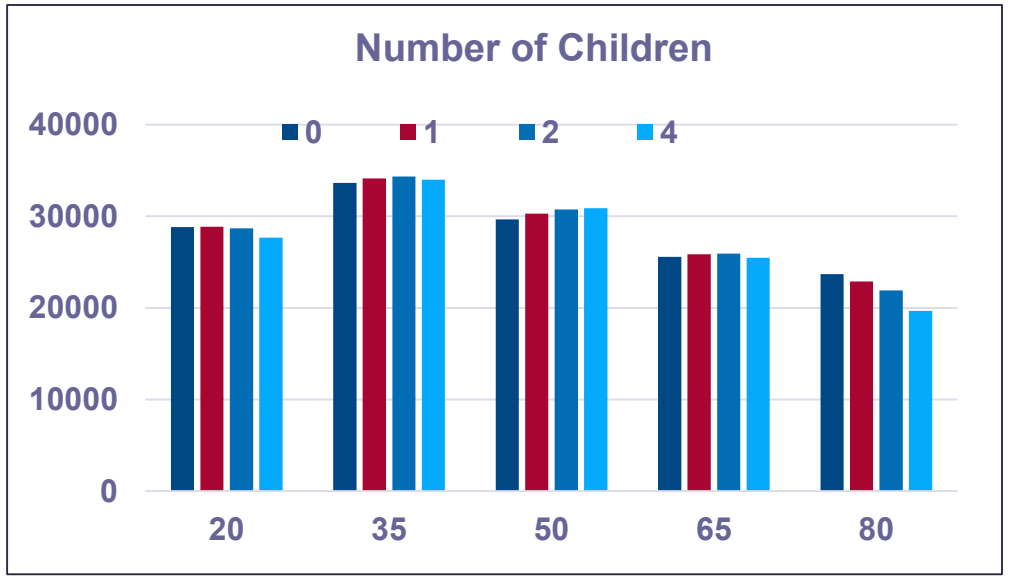


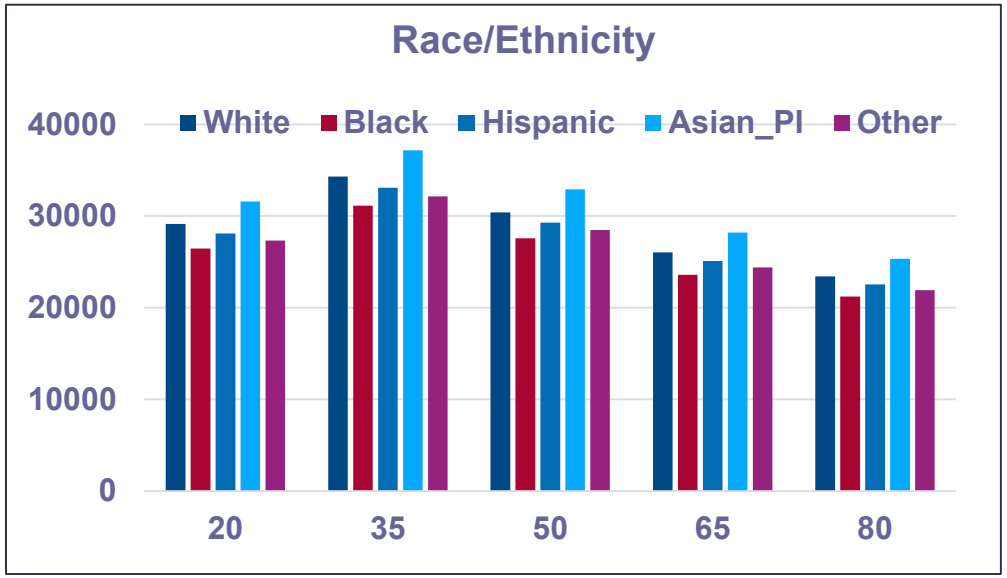


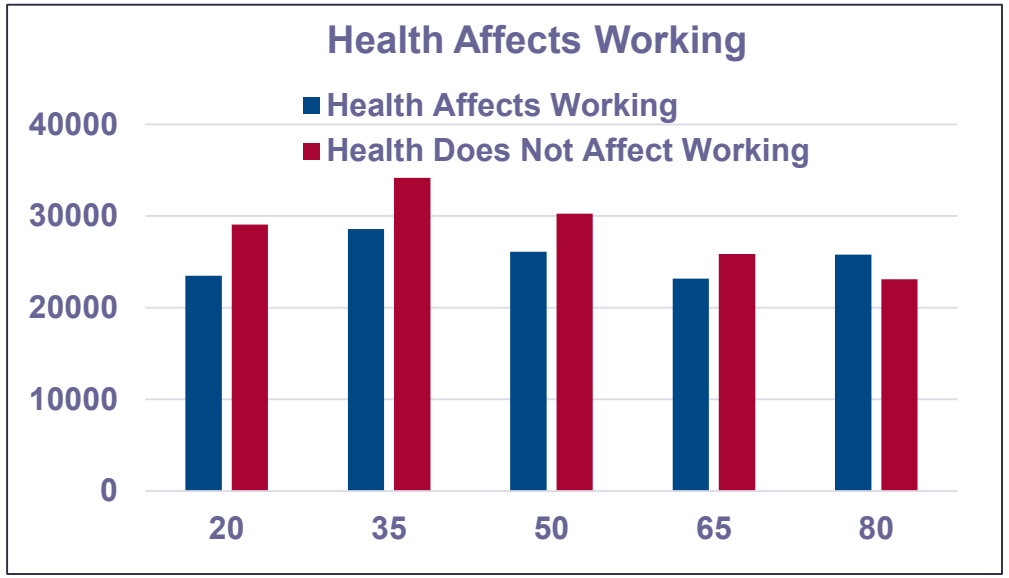


Conditional Productivity Differences by Specific Attribute at Selected Ages

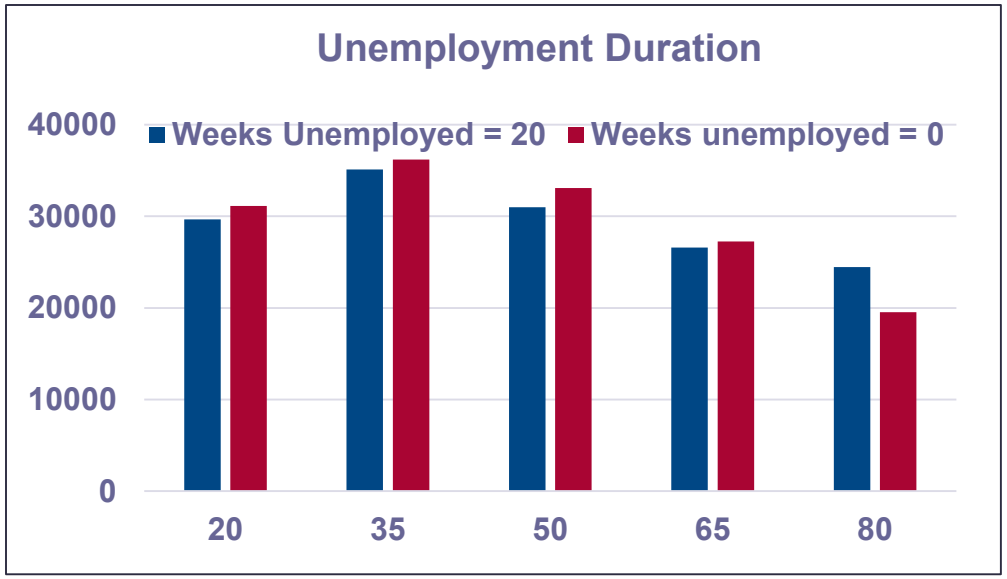


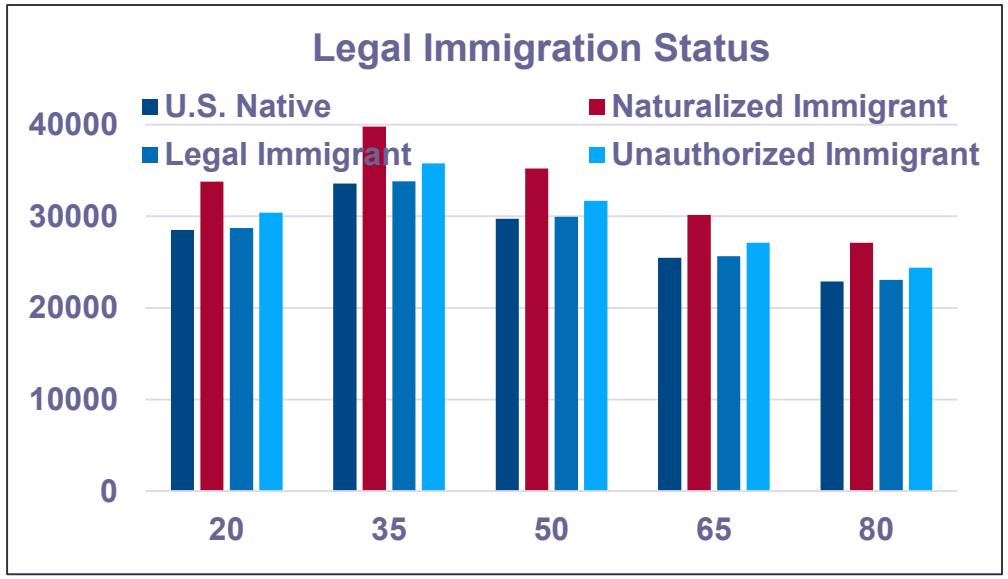


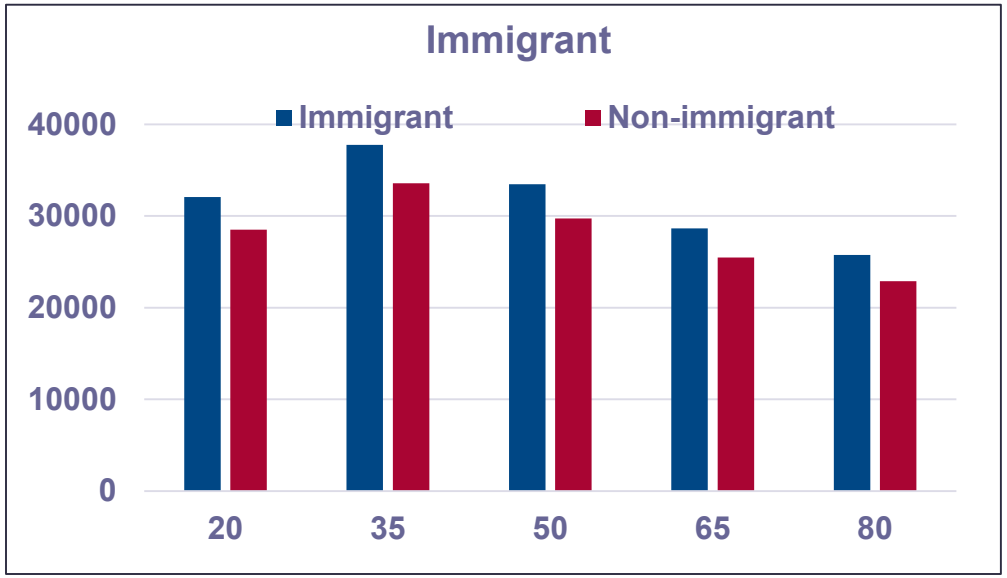


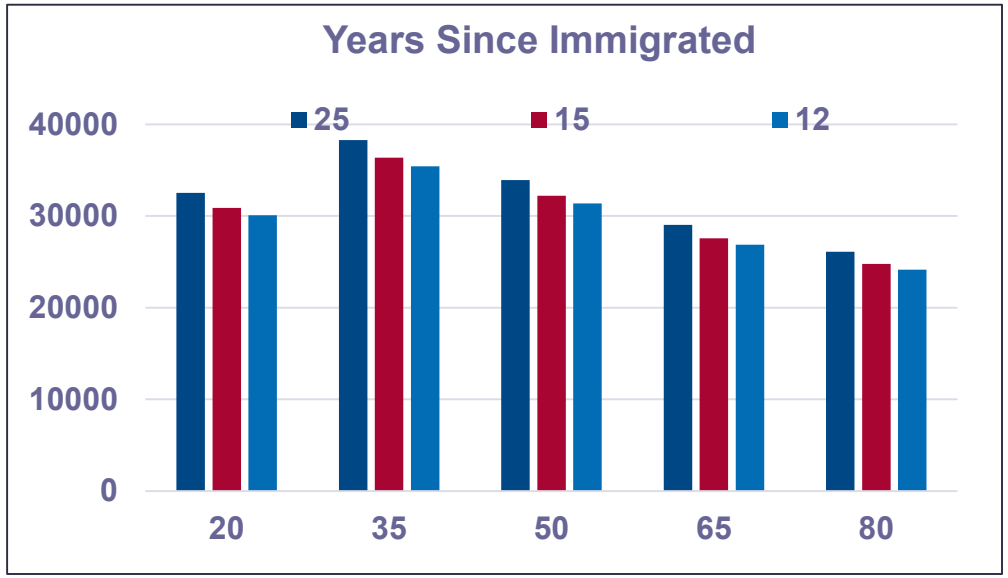


Conditional Productivity Differences by Specific Attribute at Selected Ages









Labor Productivity Growth Decomposition

Output Growth Decomposition

The production function framework implies the output growth decomposition

$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + \alpha\frac{1}{K_t}\frac{dK_t}{dt} + (1-\alpha)\frac{1}{h_t}\frac{dh_t}{dt} + (1-\alpha)\frac{1}{L_t}\frac{dL_t}{dt}$$

Each component on the right-hand-side of the equation above contributes to total GDP growth (the term on the left-hand-side).

Labor productivity growth then equals

$$\frac{1}{Y_t}\frac{dY_t}{dt} - \frac{1}{L_t}\frac{dL_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + \alpha\frac{1}{K_t}\frac{dK_t}{dt} + (1-\alpha)\frac{1}{h_t}\frac{dh_t}{dt} - \alpha\frac{1}{L_t}\frac{dL_t}{dt}$$

2018-27: 1.674 – 0.795 = 0.615 + 0.345*1.864 + 0.655*(-0.166) – 0.345*0.795

0.88 = 0.88

2018-37: 1.494 – 0.570 = 0.640 + 0.345*1.753 + 0.655*(-0.247) – 0.345*0.624

0.87 = 0.87

2018-92: 1.686 – 0.510 = 0.658 + 0.345*1.943 + 0.655*0.028 – 0.345*0.510

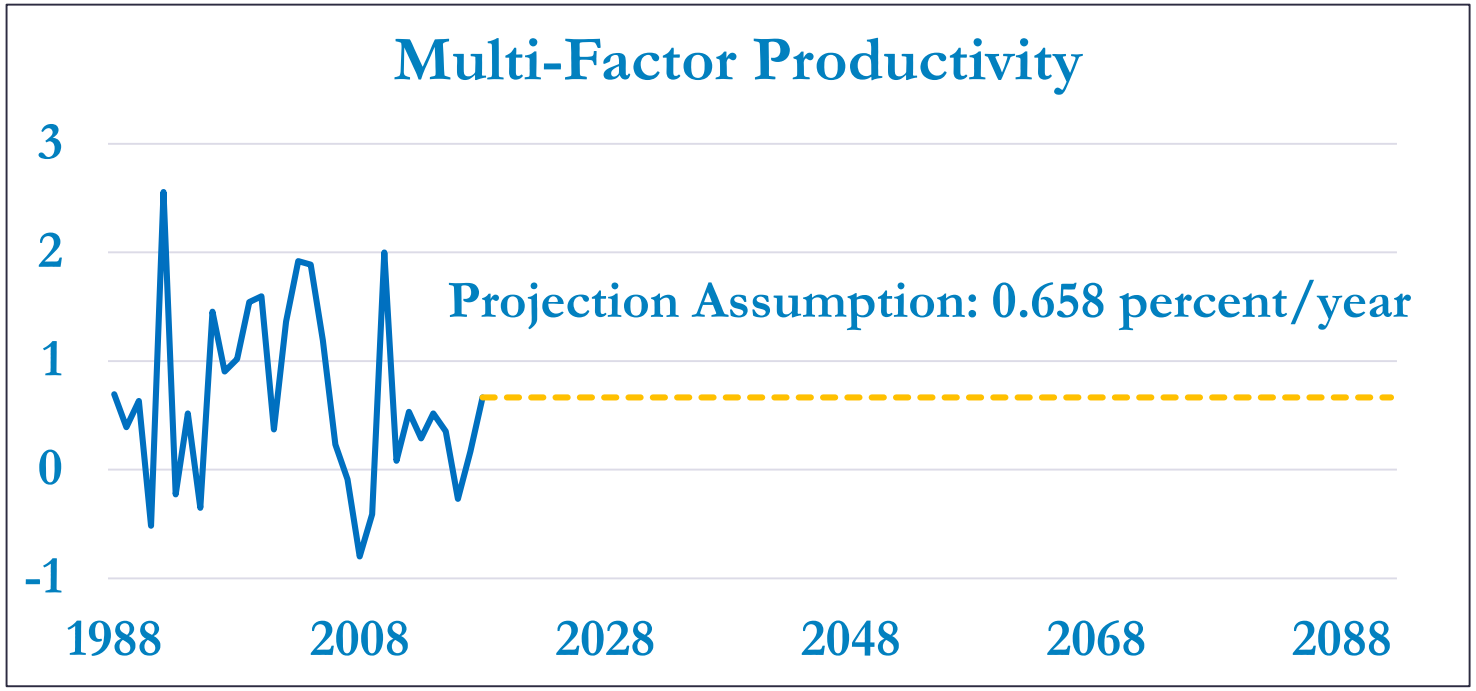
1.17 = 1.17

Production Function: Multifactor Productivity

Multifactor Productivity (A) Growth Rate

BLS Private Business Sector Average Growth (1988-2017)..... 0.86

Convert to Total Economy Basis (OO-Housing/Government/Non-Profit)..... 0.66

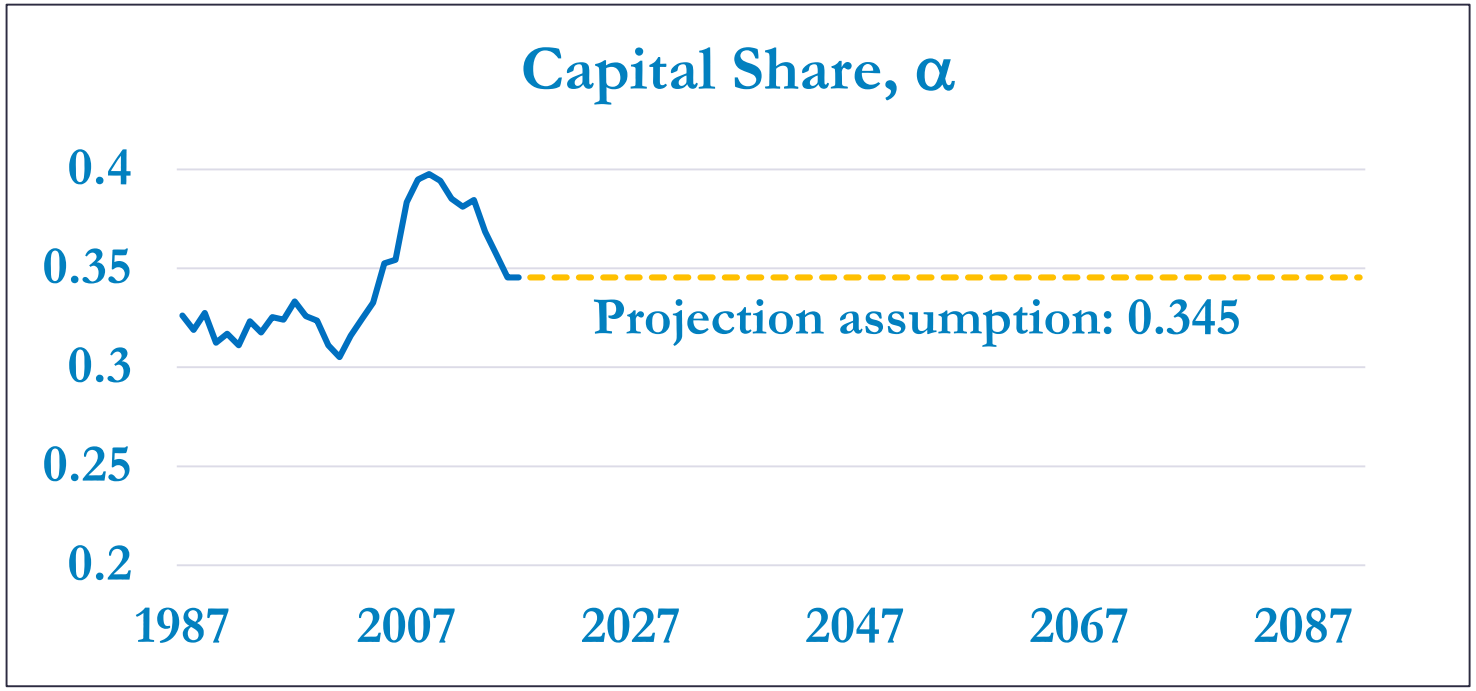


Production Function: Capital Share

Capital Services Share (α)

BLS Total Economy estimate includes extra income items

Remove net income of Government/Non-Profit from capital income and find share of GDP..... 0.345



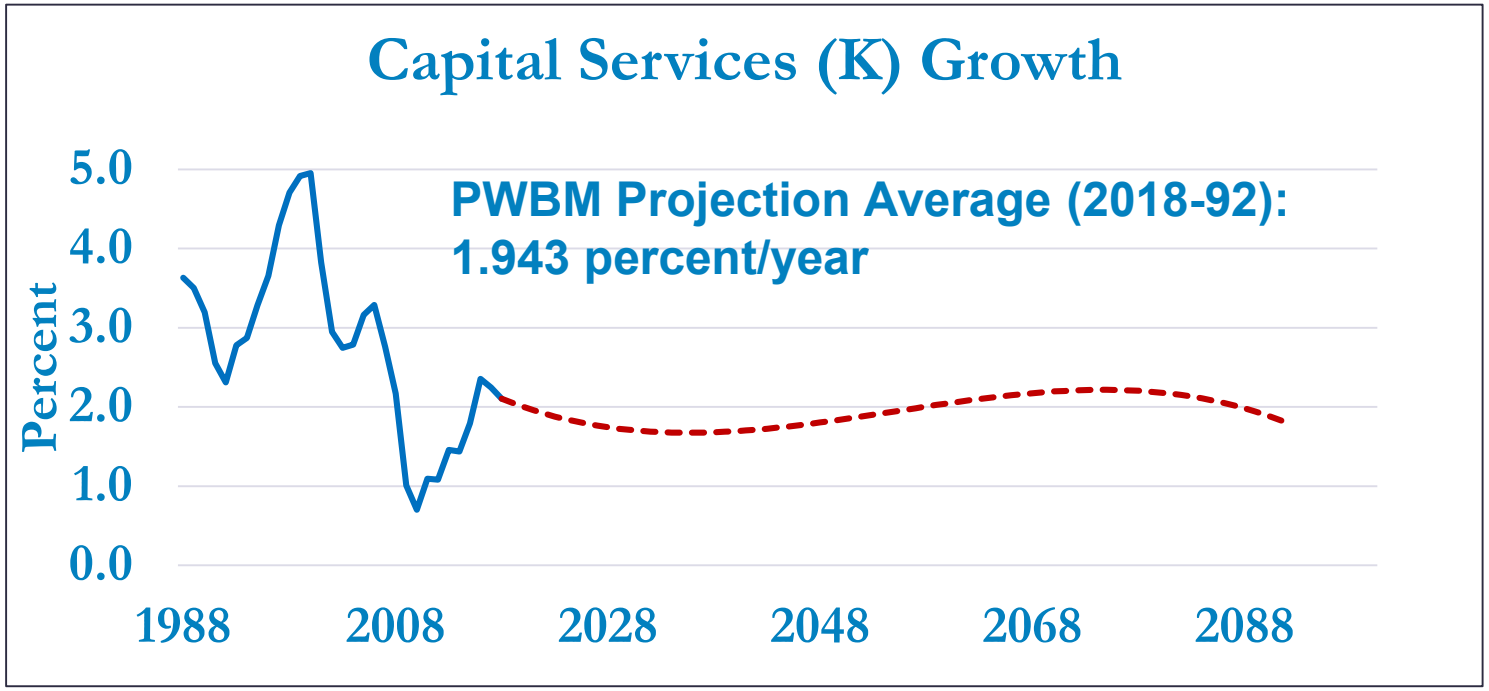
Production Function: Capital Services

Capital Services (K)

BLS: Industry specific investment history + depreciation → current stock

Depreciation rates: Differential rates of service release by short- and long-lived capital

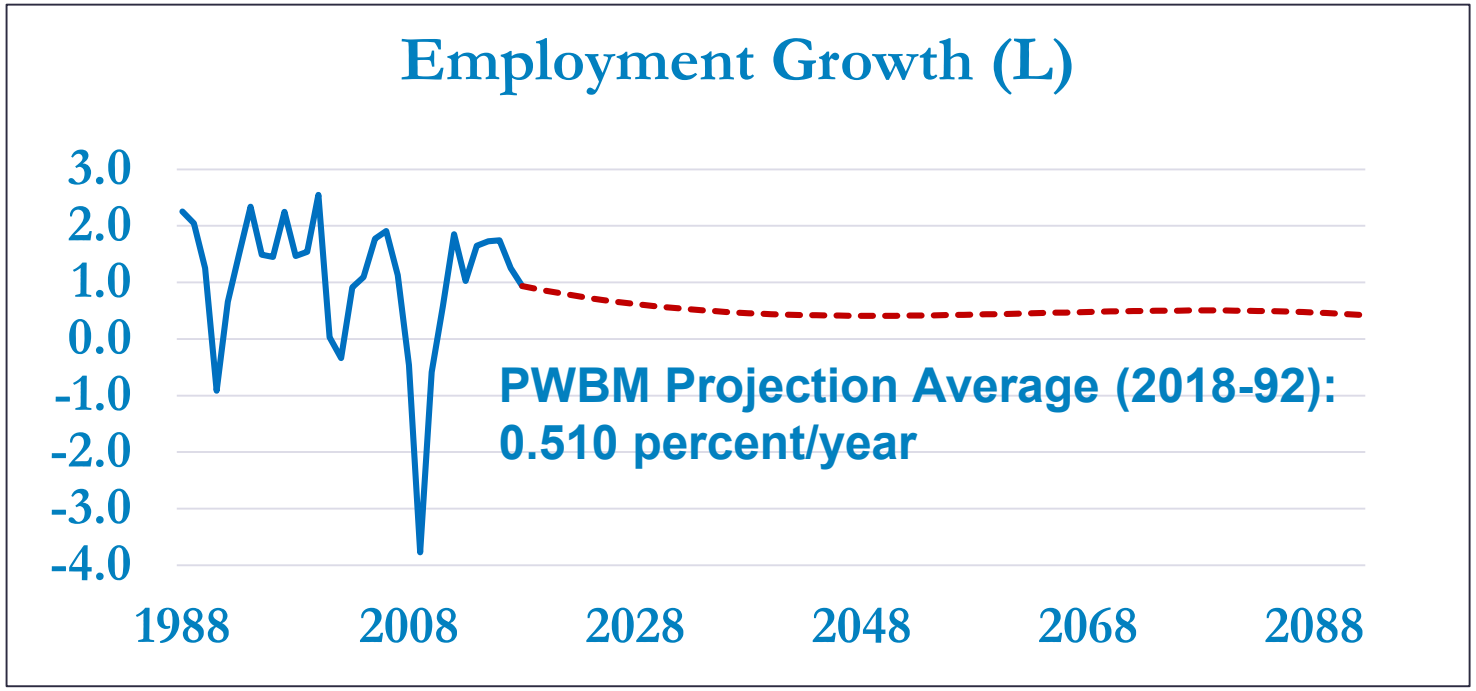
Investment rate: Capital service input assumed to grow with labor input – the United States is assumed to remain open to trade and capital flows



Production Function: Employment growth

Worker head count based on work choices correlated with projected person attributes

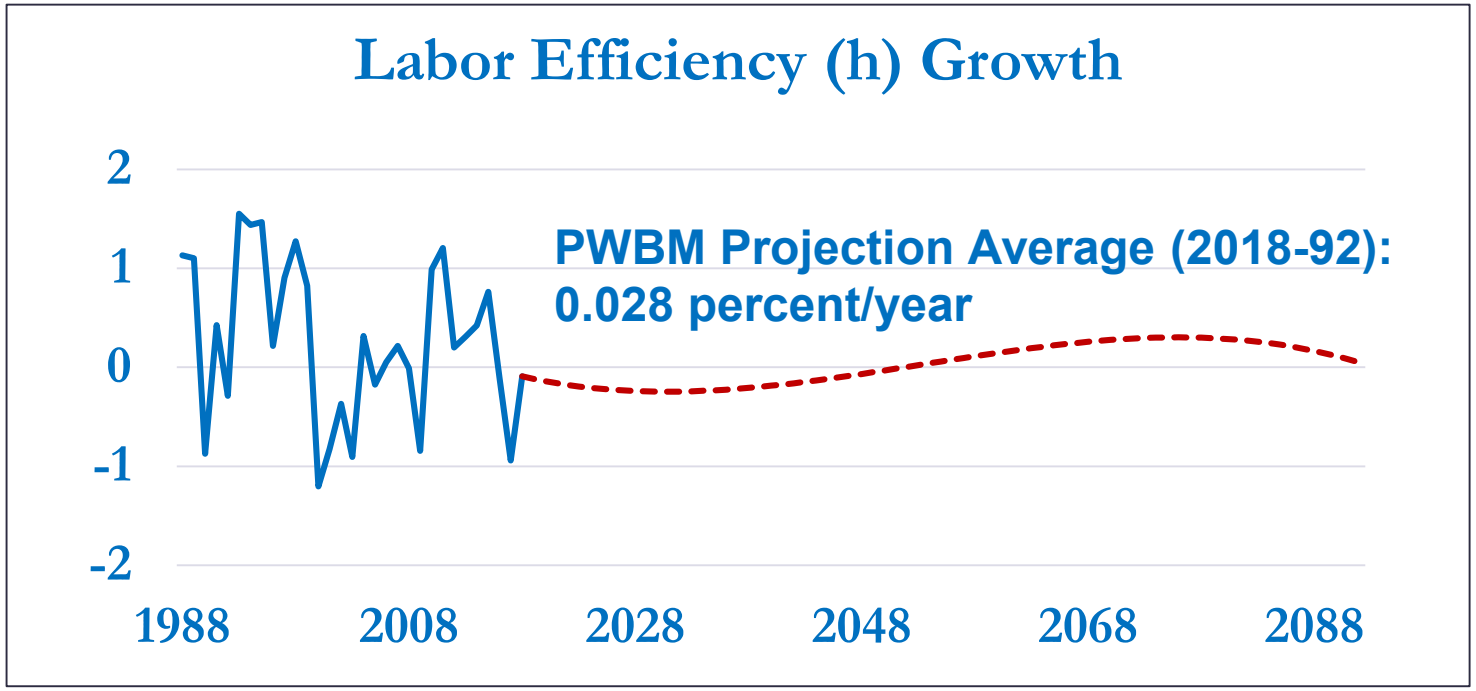
PWBM microsimulation projections



Production Function: Labor efficiency growth

Worker productivities correlated with projected worker attributes conditional on work choice

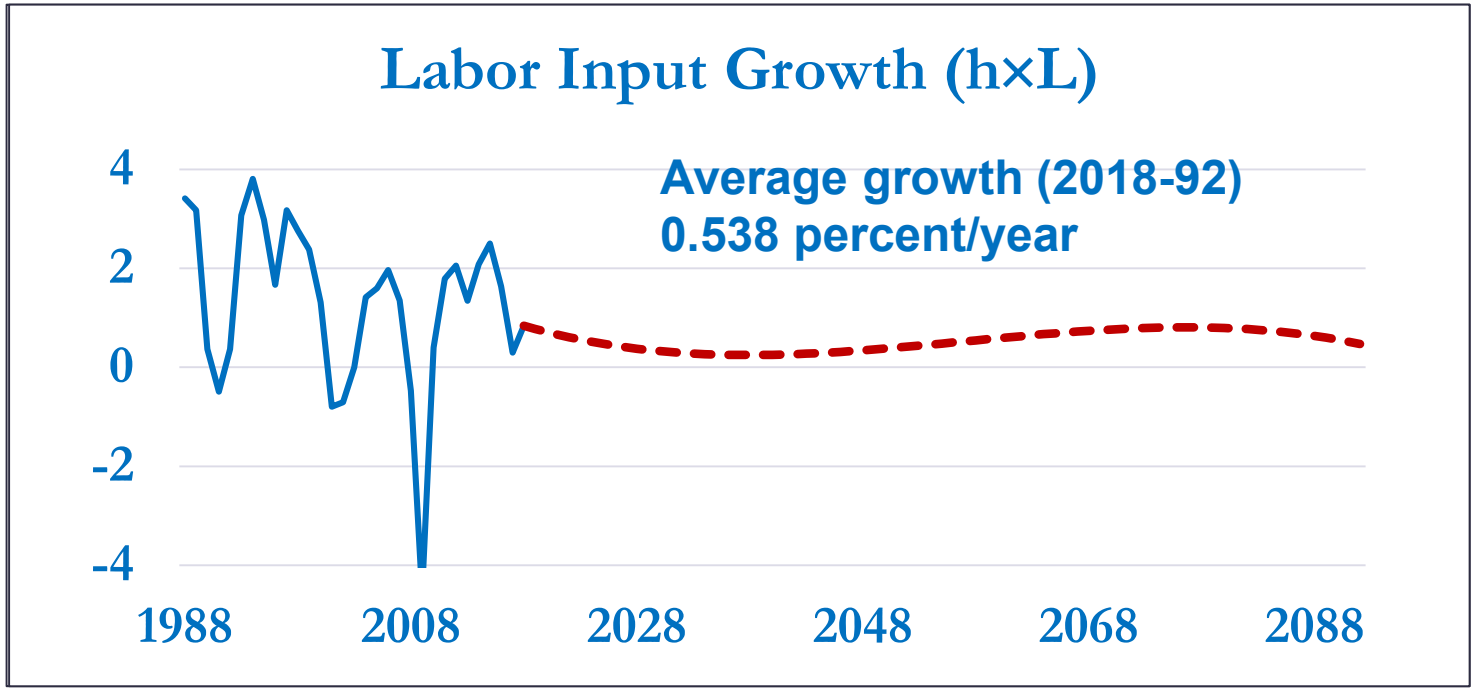
PWBM microsimulation projection



Production Function: Labor input growth

Efficiency adjusted labor services

PWBM microsimulation projection



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Budget Model